The Exploits of Joel Bennhall,

Chrononaut and Scholar

as recorded by

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Part I

Chapter 1.

Bennhall's Origins and First Four Decades

Joel Bennhall was born in Dublin in 1870. His father, Sean McDermot Bennhall, an engineer and amateur mathematician, had known Hamilton in his youth and had heard from Hamilton himself the story of his revelation in 1840 of the laws governing quaternions and his inscription of the same on Brougham Bridge. Bennhall's mother, Daniella Sophia Wilde Bennhall, a talented pianist and a passionate advocate of Irish independence, was a distant cousin of Oscar Wilde. Bennhall recalled meeting Oscar in 1887 on the occasion of a rare visit by his mother's famous cousin. While acknowledging Oscar's brilliance, Sean had always regarded the man as fundamentally unsound, and discouraged his wife's efforts to renew contact with him.

Bennhall's name "Joel" arose from an observation of his father's. Sean was by nature a pessimist, and Sophocles' dictum "Not to be born is, past all prizing, best" he took as axiomatic. His pessimism had been dispelled on meeting Daniella when he had been swept away by her zest, her beauty, her irreducibility, her otherness. But after a year or so into their marriage, Sean's pessimism resurfaced with Daniella's sudden announcement of her pregnancy. Sean's response was to quote the book of Joel, "The sun and the moon shall be darkened, and the stars shall withdraw their shining. " So it was that the child came to be christened Joel. But that was merely an irony, rather than a malediction, on Sean's part, since he came to delight in his son. Nevertheless, in his later years he reverted to pessimism, leading to his suicide in 1891. He is known to have observed, long before Jean-Paul Sartre, that the greatest gift a father can give his son is his early death. And, in accordance with his principles, he followed through.

While the young Bennhall cannot be said to have been a child prodigy in the received sense, he quickly showed himself to possess something more than the considerable gifts – linguistic, musical, mathematical - his accomplished parents regarded as his birthright. On his seventh birthday he delighted his mother by sitting down at her Blüthner and launching into a passable rendition of Beethoven's *Für Elise.* Around that time he startled his father by asking him, right out of the blue, "Daddy, why doesn't dy/dx simpify to y/x by cancelling the d's?". His father - unwilling, and, for all his authority as paterfamilias, probably unequipped to embark on a deconstruction of the intricacies of the differential calculus - was stymied. But later his mother rose to the occasion when young Joel stopped the conversation at dinner by suddenly piping up, as dessert was served, that the word "desserts" spelled in reverse was "stressed". In his unpublished (and as he would ruefully observe, unpublishable) memoir *Total Oblivion* he records, with a lingering pride in his mother's perspicacity, that she, refusing to be upstaged by a mere child, retorted, "But, dear Joel, there's no need to invoke the plural, since after all the reverse of 'dessert' is 'tressed', and am I not?" This incident, mercifully, was not to lead to Oedipal fixation in the young man, but to his enduring obsession with palindromes and the bonds of language. His later speculations on the nature of memory and his numerous unpublished works on the nature of reality can be traced to the same source.

He was privately educated by a series of tutors. In a letter to his father one of these observed: " I enjoy teaching your son, whose intelligence and enthusiasm would delight any pedagogue. But it seems to me that - probably in an effort to impress you and your wife, as well as himself- he is actually working beyond his native intellectual ability." After his father's death, he came across this letter and was considerably disturbed by it, leading eventually to his consulting Freud in 1909.

In 1884 his parents, recognizing his musical ability, sent him to Copenhagen to study music with the aging Danish composer Dag Henrik Esrum-Hellerup. In *Total Oblivion* he later reflected: "When I started my studies with Hellerup I must have been about 14, and he was well into his eighties. In those days children were brought up to revere their elders and I was no exception. I certainly respected Hellerup, and not merely for his advanced age. Nevertheless, I had begun, with adolescent pessimism, to wonder what was the *point* of aging? This may have been triggered by my learning of John Cay's epitaph: *Life*'s *a jest/And all things show it/I thought so once/And now I know it.* Youth can grasp the pointlessness of aging, even if the concrete details are beyond it: no special capacity is required, just a certain inherent pessimism which, like one's eye colour, is a permanent feature of one's constitution. On the other hand, the fact that life furnishes no cosmic revelations, no gaudy epiphanies—the fact, in other words, that one's pessimism is finally confirmed—shows that this "pessimism" is absurd, just as a child's doubts that Santa Claus will turn up at Christmas are absurd (although these are touching). To expect nothing should not lead to pessimism! In any case one can have experiences resembling epiphanies—*moments musicaux*, for instance—which, while falling short of gaudiness, at least serve to moisten the eyes. But, of course, these have only local significance, since, in the final analysis, locality is all."

In 1888 Bennhall went up to Trinity College, Cambridge following the award of a scholarship to read mathematics. There he met and befriended "Bertie", his fellow undergraduate Bertrand Russell, whose intellect and wit made a deep impression on him. He underwent what he later termed "the customary undergraduate depression". As he reported in *Total Oblivion:* "Each of my days contained a surfeit of hours. Every morning the clear sky of returning consciousness would become quickly clouded by the certain knowledge that I would soon be staring into the void, unrelieved by a providential knock on the door by Bertie, who, with his talent to engage, would, I thought with a stab of envy, be engaging elsewhere. Out of sheer self-pity I would now and then succumb to this feeling of abandonment by lying face down on the floor of my rooms in Great Court. Normally I would soon become bored with my self-posturing, pull myself together, and rejoin the outer fray. But on one occasion I was still lying prone on the floor when the gyp for my staircase, one Tubbs, suddenly entered the room and stumbled over me. 'Blimey, young sir,' he exclaimed. 'There you are, flat on the floor, and as far as I can tell, you ain't even drunk.' Which of course I wasn't - just trying to overcome *ennui."*

Despite these self-indulgences Bennhall managed to graduate, in 1891, as 11th Wrangler in the Tripos examinations. This result disappointed him, however, since it failed to live up to his private motto *aut Gauss, aut nihil*! and he quickly abandoned the idea of an academic career. The arrival a little later of the substantial inheritance from his father's estate triggered in him the notion of taking up the role of wealthy dilletante. In its pursuit he moved, first to London, then to Paris. In London he lodged at the house in Richmond of a family friend, later immortalized as the unnamed Time Traveller in H. G. Wells' 1895 novel *The Time Machine.* Bennhall, who figures in the novel as the Very Young Man, was actually present when the Time Traveller successfully demonstrated the working model of his time machine to his dinner guests. In the novel, the Very Young Man expresses great enthusiasm for the Time Traveller's achievement, and quickly grasps its potentialities. Some years later he was provided with the opportunity to realize these potentialities for himself.

In 1892 Bennhall moved to Paris, taking up residence in an apartment on the Île St.-Louis. A few months after his arrival he attended a gathering of *avant-garde* intellectuals at which he was introduced to the writer Joris-Karl Huysmans, famed for his novel *A Rebours*. In *Total Oblivion* he reports: "I had just read *Là Bas*, Huysmans' recently published novel about Occultism in Paris. It had made a great impression on me and I was delighted to have the opportunity of discussing it with the author himself. I told him that when I finished the novel I was eager to learn of the further adventures of its central character, Durtal, making the obvious hint that he should write a sequel. He replied that he was already planning one. In fact he came up with four. The second of these sequels, *La cathédrale*, published six years later, turned out to his most commercially successful work, enabled Huysmans to retire from his civil service job and live on the royalties."

In Paris Bennhall also befriended the young Paul Valéry, with whom he was delighted to find that he had much in common. In October 1892 he accompanied Valéry on a trip to Italy to visit relatives. In *Total Oblivion* Bennhall picks up the story: "On the night of 4 October I was woken up by an anguished Paul, who was clearly undergoing some kind of existential crisis. In an effort at calming him, I proposed that we discuss his difficulties over a cognac. After a while I said that he needed to devote himself to *la vie de l'esprit*. I went on to suggest, not entirely seriously, that an effective way of doing this might be for him to rise at around 5 o'clock each morning and jot down his waking thoughts in a journal. This would afford him the right to be stupid for the remainder of the day. I was gratified that he immediately took up my suggestion and adhered to it for the rest of his life. This eventually led to the publication of the famous *Cahiers.*"

In *Total Oblivion* Bennhall records his brief, but memorable meeting with Henri Poincaré. Bennhall remarks that the great mathematician's observation that mathematics is the art of calling different things by the same name might prompt the poet to claim poetry to be the art of calling the same thing by different names. Bennhall admits to being crestfallen when Poincaré replies that he'd already heard that *aperçu* from Valéry.

In October 1900 Bennhall visited his distant cousin Oscar Wilde – a little while before the latter's death at age 46 - who had been reduced to confinement in a shabby Paris hotel. In *Total Oblivion* he reports that he succeeded in persuading Wilde to venture outside his prison to partake of an *apérifif,* over which Wilde declares, with (as he describes it) "ironic despondency" *My wallpaper and I are fighting a duel to the death. One of us has to go.*

After spending a dozen or so years in Paris, Bennhall tired of merely gratifying his senses. He resolved to press the musical talent of his youth into an effort to re-establish his nontriviality in his own eyes. Accordingly early in 1904 he travelled to Vienna to study composition with Schoenberg , where he met fellow-students Berg, Webern and Wellesz, Schoenberg's brother-in-law Alexander Zemlinsky, Gustav Mahler and his wife Alma, the child prodigy composer Erich Wolfgang Korngold, and the pianist Artur Schnabel. He was struck by Berg's urbanity, charm, self-irony and noted his marked facial resemblance to Oscar Wilde. He reports in *Total Oblivion:* "One evening I told Berg about my final meeting with Wilde, and Wilde's wallpaper declaration, to which Berg responded: 'How sad that such a genius should be reduced to such degradation. But I hope that, should I eventually find myself in such dire circumstances, I would be able to make light of them so wittily.' Many years later I learned that Berg had acquitted himself admirably in that regard. For a few days before his death of septicemia in December 1935 at the age of 50, he had received a blood transfusion, which resulted in a temporary improvement. He asked to meet the donor, an ordinary young Viennese, in order to thank him, remarking to a friend, who later became his biographer, 'I hope I don't turn into a composer of operettas now!' In *Total Oblivion* Bennhall also notes a certain obsessiveness in Berg's character: "one evening, over a glass of the *Niersteiner* to which Berg was partial, he confided to me that the he had developed what he described as 'an unhealthy fascination' with the number 23. I could not resist telling him, tongue in cheek: '23 is entirely harmless, an unremarkable number whose primality is its sole distinction. I should be seriously concerned, though, if you had told me that you had become partial to the number 37. Over and above its primality, that number has the disturbing property of being a factor of 666, the number of the Beast.' I was surprised at his response: 'Yes, it's true, I was initially attracted to 37, but I soon saw that it possessed that property , and so was best avoided. I'm glad to hear that you regard my new numerical obsession with 23 as harmless.'" Berg later produced a number of compositions based on the number 23, including the *Lyric Suite.* In *Toral Oblivion* it is reported that hat "Berg had been struggling throughout 1907 - 08 to finish his Piano Sonata (which was to become his opus 1). He had completed the first movement but, he was 'bereft of inspiration' as to the composition of the two further movements he had originally envisaged. I was struck by the beauty and originality of the one movement he had completed, and I had to acknowledge a certain envy at his achievement. I told him that I felt certain it could stand alone, observing that the truncation of Schubert's *Quartettsatz* and *Unfinished Symphony* has not impeded their progress in the musical world. Schoenberg, characteristically, later put the matter more bluntly, telling Berg that his inability to complete the work showed that he had said all he had to say." While deeply impressed with Schoenberg's indomitable spirit and passionate conviction as a composer, in a letter to one of his Irish cousins Bennhall wrote "I find Schoenberg something of a martinet, issuing orders to his students in the manner of a drill-sergeant. In his eyes, to disobey an order amounts to nothing less than a betrayal. He requires disciples, not students." Nevertheless, Bennhall remarks in *Total Oblivion* that Schoenberg's great expressionist works *Three Piano Pieces, Op. 11* and *Five Pieces for Orchestra, Op. 16,* "came as a total revelation to me, as indeed they did to my fellow-students.`` The impact of these works inspired him to compose his *Seven Episodes for Eleven Instruments* and *Chromatic Miniatures for Solo Harmonica,* works he later dismissed as "not merely unplayable, but not worth playing."

Bennhall goes on to report that, during a tutorial held sometime in 1906, Schoenberg suddenly exclaimed: "To make progress in music you must feel as if you have suddenly been plunged into a bathtub of boiling water. If you do not feel this, you will never produce anything but kitsch!" Many years later he recalled this remark on learning of Schoenberg's account, in a 1947 speech in Los Angeles, of his life as a composer: "Personally I had the feeling as if I had fallen into an ocean of boiling water, and not knowing how to swim or to get out in another manner, I tried with my legs and arms as best as I could. I do not know what saved me; why I was not drowned or cooked alive. I have perhaps only one merit: I never gave up! But how could I give up in the middle of an ocean...." In reporting his reaction to this Bennhall writes in *Total Oblivion* : "Poor Schoenberg: out of the bathtub and into the ocean! But I admire his courage since I myself have done little more than dangle a toe in that vast cauldron. Nevertheless even my hesitant probings have served to confirm to me the existence of Schoenberg's boiling ocean, along with its temperature and turbulence."

In 1909 Bennhall consulted Freud about the feeling of intellectual inadequacy which had come to haunt him ever since it had entered his youthful mind through his reading of his tutor's letter to his father. As he records in *Total Oblivion*, "Freud ushered me into his consulting-room, which I saw was equipped with the legendary couch already well-known to select Viennese neurotics. He immediately asked me, with much the same directness I had come to expect from Schoenberg, ' So what can I do for you?' I told him, with what I felt was equal directness, that I had never been able to overcome the fear that I was 'working beyond my ability.' I was startled, and then delighted at his response. With a wave of his cigar, he said, 'You should regard yourself as lucky. At least you actually possess some ability to work beyond. And in any case those who are not truly working beyond what others characterize as their native ability will produce nothing but platitudes' Freud's trenchant observation instantly banished my neurosis."

Chapter 2

Bennhall Becomes a Chrononaut

Early in 1910 Bennhall returned to London, taking up residence in a large house on Primrose Hill Road facing Regent's Park. On the morning of 18 May, a few months after he had settled into his new lodgings, a parcel, in the form of a shallow box, arrived for him in the post. Approximately 9 by 12 inches in size, the box proved to contain a number of handwritten sheets and, encased within a satin pouch, a slim flat rectangular, curiously elegant object, just large enough to fit snugly into its receptacle. The object's edges were studded with a number of small buttons of various colours, each of which bore what appeared to be an embossed label. Both back and edges were metallic, but what he took to be the object's front was composed of a transparent glass-like substance, giving it the appearance of a window looking out onto a pitch-dark night. Bennhall had never before seen anything remotely like it. His familiarity with the scientific romances of H.G. Wells immediately brought the term "futuristic" to his mind. Intrigued, he unfolded the handwritten pages also enclosed in the box. These proved to be a letter, of considerable length, from the Time Traveller, the family friend with whom he had lodged on his first stay in London several years before. It read as follows.

15 May 1910

My dear Bennhall,

It will no doubt come as a shock to receive a letter from me, five years having passed since both I and my Time Machine vanished without trace. But that shock will surely fade into insignificance in the face of the importance of what I have to tell you, and the power of the device I have sent you.

After my sudden disappearance, I understand that there has been much speculation as to what has become of me. Many seem to have regarded the whole business as a hoax or "publicity stunt" (although it's hard to see what its purpose might be); others thought that the machine itself had somehow disintegrated, taking me with it. Those very few, including, I believe, yourself, who took seriously my claim that I had constructed a mechanism that would truly carry one through time, became convinced that I had actually left on another temporal journey.

I well remember that dinner party in Richmond at which I first discussed the possibility of travel in the fourth dimension, and put my model time machine through its paces. The enthusiasm and receptiveness to new ideas you showed on that occasion was in such refreshing contrast to the scepticism of the other guests, and especially to the stubborn unimaginativeness of that idiot Filby. I have often wondered why I invited him in the first place! But you quickly grasped the idea of building a machine which would make voluntary movement in the fourth dimension possible, and you recognized that, in effect, it would enable one to travel in time. You remarked astutely that, through financial investment in the present, the machine could be used for making a handsome profit in the future. It is the acuity and enthusiasm you displayed that memorable evening which have emboldened me to send you the mechanism that you are no doubt puzzling over, along with the present missiv.

Let me now tell you of my spatio-temporal travels (yes, both in space and time) since my "disappearance" . My initial journeys into the far future had revealed it to be such a bleak and depressing place that I had no wish to return there. Accordingly I resolved to travel into the past. Not knowing how long I would remain in the past, I converted a portion of my financial assets into gold bullion which I had good reason to believe would be acceptable as currency for purchase of the necessities of life in whatever circumstances I might find myself. When the time came for my departure, I loaded the bullion into the machine, set its levers appropriately , and embarked on my temporal Odyssey into the Past. I made a number of stops of lengths varying from days to months, during each of which I was fortunate in finding somewhere to conceal the machine, freeing me to venture into the London of past times. The further back in time I went, the smaller and more squalid the city became. I witnessed the dreadful toll taken on Londoners by the Great Plague of the 1660s, and the devastation caused by the Great Fire of 1666. But I also saw the splendid effort made soon afterwards to rebuild St. Paul's Cathedral and the construction of its magnificent dome. I was even able to attend a few meetings of the Royal Society, actually being fortunate enough to witness the great Newton himself presiding at one of them. I travelled as far back as Roman times when London, which they called Londinium, *was no bigger than the Hyde Park of our day.*

This fascinating journey into London's past, and all the stops I made there, occupied a good ten years of my life. But eventually I tired of the inconvenience of having to conceal the time machine on each occasion I left it. I had originally conceived and built it as a kind of carriage conveying its occupant through time. I saw that it would be much improved if its size could be reduced to that of a book which I could carry on my person. The engineer in me became fascinated with this conception for which I have coined the unlovely term "miniaturization". I had not the slightest idea how miniaturization of my machine could be achieved. Then it struck me that, given the increasing pace of scientific progress in my own time, the new technological developments necessary for achieving my goal might well emerge within the coming century. It was also clear *that to build my miniaturized machine would require a great deal of money, of a magnitude vastly exceeding the relatively modest outlay I had made in constructing my original machine. At that point I recalled your observation that, if one could travel into the future, a profit could be made by investing one's money at interest, and, as you put it "hurry on ahead". And this is precisely what I did. I used the machine to return to a moment not long after the instant of my "disappearance" and quietly invested all my remaining funds, at a decent rate of compound interest, in a solid financial firm.*

Then I moved forward in time, through the 20th century. I saw the emergence of revolutionary new scientific theories, relativity and quantum theory, overturning our old Newtonian conceptions of space and time. I witnessed the destructive release of the power of the atom in a devastating world war in the middle of the century, and the subsequent development of that power into a monstrous sword of Damocles threatening the destruction of the entire world. I saw the emerging evidence of the heating of the atmosphere and the rising oceans caused by the accumulation of carbon dioxide through the excessive use of fossil fuels, and humanity's failure to curb it.

But in my obsessive quest to refine my time machine, I focussed my attention on two related technological developments, computing machines – known universally as "computers" - and electronics. Bear with me while I give you a brief summary of the evolution of these fields.

The field of electronics was launched by the identification of the electron by Thomson in 1897, along with the invention in 1904 of the vacuum tube which enables small electrical signals to be amplified, and can be used as an electrical switch. The tremendous future significance of electronics was revealed by my journey into the coming century. It would be no exaggeration to say that electronics will become the single most important influence on the society of the future. Indeed future historians will term the 20th century the "electronic age". In the mid-twentieth century the comparatively bulky vacuum tube will be superseded by the compact transistor and, crucial to my purposes, the miniaturization of transistors to the size of molecules.

As for the computer, its origins go back to the ancient abacus. Even in our day computers are still mechanical devices of one sort or another. In my journey into the near future I saw the emergence of the electronic digital computer in which computations are based on digital, binary arithmetic and the use, first of vacuum tubes, and later of transistors, to provide physical embodiment of the computation process, replacing the gears and cogs of mechanical devices. This development will permit a vast increase in the speed in which computations could be carried out as well as an extraordinary reduction in the size of computers themselves. Digital computers, used primarily *for the processing of information, will have such an impact on everyday life that the period of their introduction will come to be called the Digital Revolution.*

I became convinced that these future technological developments could furnish the means for miniaturizing my time machine. I was also excited at seeing that the remarkable advances in theoretical physics in the 20th century might make it possible for my machine to travel in space as well as in time, that is, in four-dimensional spacetime. By the year 2020, I judged that all the practical and theoretical elements necessary to launch my project had become available. Accordingly I set up my base of operations in that year. By that time my financial investment in 1905 had swollen to several hundred million pounds, so I proceeded to set up an advanced technological facility and employ the best contemporary scientific talent to assist me in the pursuit of my project. We became a kind of sodality of dedicatees to the programme, like the ancient Pythagoreans, all sworn to secrecy. It took a good decade of hard work, and the overcoming of numerous setbacks, for us to achieve our ultimate goal: the fashioning of a portable device capable of carrying its bearer to any chosen location and time on the Earth's surface.

In addition to the engineering difficulties we encountered in the actual construction of the device, our little group had to think carefully about the problems that could arise with travel into the past, and more especially with attempts to change it. We could not see any fundamental problems arising with past time travel in which no attempt is made to change the past, although certain subtleties emerge. Suppose, for example, that I travel to June 1813 to witness the Battle of Waterloo. If my sudden materialization from the future were to be noticed by a number of startled soldiers on the battlefield and they later report the incident, this fact has entered the historical record at the time I make my trip into the past. My trip into the past doesn't alter that past, since my appearance on the battlefield is already a past event before the trip takes place. And if in the end I decide not to make the trip, then I simply don't appear on the battlefield at all. Accordingly in neither case is there an alteration of the past. But, if I do in fact make the trip, it retroactively causes my appearance on the battlefield in the past. This is an instance of reverse causation in which an effect precedes its cause Of course, this reversal of cause and effect arises in objective historical time. From my subjective point of view the two events - my trip into the past and my appearance on the battlefield - occur in the "right" order in which the trip takes place before the appearance. My trip into the past thus induces a reverse causation in objective time – a causal loop - *which, as the time traveller, I do not experience myself. Despite its oddness, this phenomenon cannot be regarded as a genuine paradox. There is no principle of formal logic preventing an effect from preceding its cause.*

But we saw that paradox could arise when travel into the past involves attempts to tamper with the historical record, or comes into conflict with participants' memories. Here is a scenario which *frequently came up in our discussions. Suppose that at some point after the invention of the time machine, I attempt to employ it to meet myself as I was in the past, in particular at the very moment I first conceived the idea of designing a time machine. I recall that this occurred when I was sitting at my desk in my room at precisely 4 p.m. on Dec. 6, 1886. I set the machine so as to arrive in my room at exactly that time. On its activation, I am returned to that place and moment. When I arrive, I find, as I had anticipated, my younger self sitting at his/my desk. I attempt to convince him that I am the person that he will become in the future, but he, startled at my sudden materialization out of thin air, is initially unwilling to accept this. However, his scepticism dissolves once he grasps that my appearance automatically confirms that his/my efforts to construct a time machine will ultimately he be crowned with success. But is this person I encounter in the past truly my younger self? There would be good reason for me to doubt it. To see why, let me call A my younger self who actually conceives and develops the original time machine and B the fellow who is sitting at my desk when I arrive in the past. Now, I must infer that B cannot be the same person as A. For, at 4 p.m. on Dec. 6, 1886, person A, my actual younger self, is seated at his/my desk, and, as I recall, saw nothing unusual. But at that same time and place person B is startled by my sudden arrival. So, if my memory is reliable, that is, if it faithfully reflects objective reality, persons A and B have had different experiences at the same time and place. It follows that they must be different persons. Thus the person I meet after my trip into the past (B) cannot be my past self (A). Accordingly person B must be, so to speak, a copy or duplicate of my younger self engendered somehow by my trip in to the past.*

We were thunderstruck when we realized that this duplication does not merely affect my younger self, but the whole universe! For when I return to the past and meet person B, what in the meantime has happened to person A, my younger self? He was originally present at 4 p.m. on Dec. 6, 1886, but now seems mysteriously to have disappeared! It cannot be the case that he has "become" person B because the two cannot be identical. But at 4 p.m. on Dec. 6, 1886, it would seem that both A and B are sitting at the same desk. For this to make any kind of sense we could only conclude that there must also be a duplication of the desks, with A sitting at one desk and B at the other. But the two distinct desks would then occupy the same position at the same time. While this may be, just barely, logically possible, a more intelligible, if still astonishing conclusion is that the two desks must occupy different realms of existence, in short, different universes, a universe 1 containing, along with myself, the original desk with A, my younger self sitting at it; and a universe 2 containing, again along with myself, the other desk at which B sits. Thus we were forced to conclude that my journey into the past causes the original universe to split or branch into two. My younger self, person A, occupies universe 1 until I travel to universe 2, at which time I vanish from universe 1, while my younger self remains in the past of universe 1. This, in a nutshell, is how time travel can produce the duplication of universes.

Then there was the Grandfather Paradox. We were already familiar with this one, since it had been previously conceived by a number of ingenious writers of what had come to be known in the 20th century as SF , science- or speculative - fiction. Let's suppose my grandfather is a domineering, unpleasant man whom I detest. But he has made a fortune in business which I have inherited and which has provided the funds for building my time machine. Nevertheless my detestation of the man is so intense, I'd still like to do away with him despite the fact that he died when I was an adolescent. Now I've built my time machine, I see that my desire to do away with my grandfather could be realized by travelling to a time when he was a young man and despatching him then. Accordingly I purchase a rifle, become an expert marksman through target practice, and travel back in time to a moment when my grandfather was still alive. I conceal himself somewhere along the route of my grandfather's morning walk, and, rifle loaded, await my victim. My grandfather appears, but in the instant I raise and aim my rifle, it suddenly flashes through my mind that, in my murderous obsession, I have completely overlooked the fact that at the time I have chosen to kill my grandfather, my father has not yet been born. I see that then to pull the trigger or not to pull it leads to munch more than my grandfather dying or not dying. If I pull the trigger, and my grandfather dies, a paradox ensues, for then my grandfather died before my father could be born, so that neither he nor I, nor my time machine ever came into existence, and as a result the whole sequence of events leading to the pulling of the trigger never occurred. In that case, the only logically consistent conclusion is that, if I pull the trigger, and my grandfather dies, the universe splits into two, one the "original" universe in which my grandfather, my father, and I pursue our lives as "before"; and the second universe in which the grandfather dies before producing any descendants, but in which I suddenly materialize at the time of his death. On the other hand, if I don't pull the trigger, then my grandfather goes on to sire my father, who in turn sires me. I go on to construct my time machine and travel into the past as before. In this case there is no splitting of the universe because I have not attempted to alter the past by killing my grandfather. To pull, or not to pull? That would then be the question.

One of us came up with the following alarming example of a potential paradox when travel into the future is followed by travel into the past. Suppose I use my time machine to travel one minute in the future, and then return one minute before I set out. I see my former self ready to travel into the future, and perversely try to prevent him from doing so. If I succeed, then a new universe is generated containing both myself and my one-minute-younger self. If I don't succeed, a different new universe is produced containing myself and my one-minute and two-minute younger selves. This can be indefinitely iterated, generating a bizarre series of universes containing an unbounded number of copies of myself.

One of the younger members of our group came up with this paradoxical scenario. Suppose I travel one week into the future and then six days into the past. I wait for six days until my former self arrives, meet him on his arrival and witness his return to the past. If this has genuinely taken place

I will encounter my future self on my arrival in the future. Thus if I do not in fact encounter my future self when I arrive in the future, I can be certain that I will not travel into the past and await my arrival in the future. So if I perversely decide to do just that, thereby meeting my past self, then that "past self" cannot be the past self who originally embarked on his journey into the future, but must be a duplicate. It follows that at or before the point at which I meet this duplicate I have entered a second universe in which the original trip into the future is undertaken not by my past self, but by a duplicate. Mercifully, this duplication of selves and universes can be circumvented if I simply do not return to the past, or I return to the past but avoid meeting my past self on his arrival in the future.

It soon became apparent to us that travel into the past would not be accompanied merely by reverse causation, but could produce curious causal loops in time. One of us, a young lady with a literary bent presented the problem in a particularly entertaining way. In the year 1599, she imagined, William Shakespeare is suffering from writer's block. The Bard is standing at his desk one day, quill in hand, but devoid of inspiration. Suddenly a hole opens up in the wall of his room and a man steps through it. He bears a manuscript which he hands to the astonished Shakespeare. It is entitled The Tragedy of Hamlet, Prince of Denmark, by William Shakespeare. *The man explains that he is a professor of literature from several hundred years hence, and that he has travelled back in time to 1599. In his time, he declares, "Hamlet" is one of the most celebrated works of literature, and he has admired it since he first read it in his youth. The professor then takes his leave and re-enters the hole in the wall; it closes, and he vanishes, leaving the manuscript in Shakespeare's hands. After the professor's departure, Shakespeare reads through the manuscript, finds it intriguing, and decides to pass it off as his own work. He never reveals its true source.*

But what, precisely, she asked, is Hamlet's true source? Our young lady suggested that there were (at least) two possibilities, one in which the universe splits, the other involving a closed causal loop. In the first of these possibilities, Shakespeare himself actually wrote Hamlet sometime in 1599 (or soon thereafter), and the professor materializes early in 1599 before the play was written. In this case, there must be two Shakespeares, the "original" one who actually wrote Hamlet and the other a temporal duplicate whom the professor actually meets. There must also be a corresponding splitting of universes into two, the first—the "original" universe— containing the original Shakespeare, and the second containing the duplicate. In this case Hamlet does not get entangled in a causal loop. The source of Hamlet is the original Shakespeare. This remains the case in the second universe, even though the original Shakespeare is no longer present in the second universe after the professor's arrival.

The second possibility, she observed, pivots on setting the whole scenario within a single universe. In this case a causal loop is generated. Here Hamlet was not written by Shakespeare through his own inspiration (in any universe), but he does actually receive the manuscript of Hamlet from the time-travelling professor in 1599, and passes it off as his own work. This scenario could, in principle at least, consistently take place without requiring the universe to split. But then, while avoiding logical contradiction, we are left with the sheer oddness of the resulting closed causal loop. For who actually wrote Hamlet? It cannot be the professor, since he first read the play in his youth. It cannot be Shakespeare himself, since he plagiarized it. Then who? Nobody, it would seem. Hamlet has materialized causelessly as the centre of a loop in time.

For readers and writers of science fiction, scenarios such as these would be regarded as no more than entertaining conceits and not to be taken seriously. We, on the other hand, bent on constructing a concrete functioning time machine, had to think in earnest about these potential snarls in time, and how to circumvent them.

We spent long hours trying to overcome these difficulties. It gradually became clear that all paradoxes leading to the duplication of universes arise through attempting to change the past, engendering contradictions with the historical record, or with the memories of the participants. Most of us thought that these exotic eventualities were best avoided, although a small group of young firebrands among us disagreed, arguing, in effect, "in for a penny, in for a pound, what's wrong with creating new universes?" I and others argued that there is already sufficient misery and pain in this universe, so that we should strive to avoid creating, even unintentionally, new realms of suffering.

But the Grandfather Paradox showed that imposing constraints on travel into the past sufficiently restrictive to prevent all possibility of paradox would make such time travel impossible. For if I could travel to any time in the past, in principle I could arrange to generate a paradox by eliminating myself at the time of my departure either by killing my former self or one of my ancestors, depending on how far I chose to travel into the past.

In the end we saw that there was no way to equip our device with a "spacetime governor", preventing its operator from setting it to travel to a point in spacetime which could generate paradoxes and duplication of universes. It could not, in in other words, be made entirely "foolproof". Free will on the part of the would-be time traveller would still obtain. We came reluctantly to accept that, used incautiously, the device could generate paradox and multiple universes. And so we vowed that, if we were to succeed in constructing such a device, we would do our best to prevent these unfortunate eventualities from coming about.

Our goals agreed, we set to work. By the year 2030, after much labour, and many setbacks, our efforts were crowned with success: the creation of a portable device capable of carrying its bearer to any chosen location and time on the Earth's surface. We decided to call it the "Space-Time Liberator" or simply the "Liberator". We produced one device for each of us, and an additional one for you, my friend, which is enclosed in the parcel I sent you. You will easily figure out how to use the Liberator's controls to set it to convey you to any point and moment on the Earth's surface you choose. Once set, the Liberator's screen will display a view of the immediate neighbourhood of your destination. It has been designed to convey to its destination its bearer and everything on his person, such as what he is wearing or holding, and including, of course, of course the Liberator itself. I have no doubt that you will use the device with discrimination. I know in advance that you will do so, because I have used my own Liberator to confirm the fact! While it's best not to use the Liberator to meet ourselves, we might well meet one another on our travels in time.

Good luck and happy journeys, your

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When Bennhall finished reading the Time Traveller's letter, at first he could scarcely believe what he had read. He accepted that the Time Traveller had travelled into the future and returned using his original machine, an event which Bennhall had actually seen. But these new developments seemed beyond the wildest fantasies of imagination! It crossed Bennhall's mind that the Time Traveller had simply taken leave of his senses or was playing an elaborate joke. On the other hand, his admiration for the Time Traveller's genius and personal integrity inclined him to believe there was a good possibility that the assertions made in the letter were true and that the enclosed device could actually do what was claimed for it.

He turned to the "Liberator" itself. It was light and sleekly elegant, altogether beautifully designed. His attention was drawn to a button on one of its edges marked *Destination.* He pushed it, and the device's screen suddenly lit up, displaying the instruction *Enter time: year , month, day, hour, minute, second* . Startled at the speed of the machine's response, he hesitated, still not taking the situation entirely seriously. Muttering "oh, what the hell", on impulse he entered a date precisely 10 days in the future. The screen then displayed a new instruction: *Enter location: choose one: (a) present location or (b) other location.* Uncertain what to do next, he noticed that the signs *(a)* and *(b)* on the screen were flashing, so, again on impulse, he pressed his fingertip directly on*(b).* The screen then presented him with what appeared to be a map of the region surrounding London, superimposed with a grid composed of a number of small squares. There was also a tiny arrow which he found could be moved with his finger to occupy any one of the squares. He moved the arrow to a square over Regent's Park. Seeing a button on the device's edge marked "Enlarge view" he pushed it and immediately the screen showed a view of the London Zoo at the north edge of the park. Spotting another button marked *Execute,* he pushed that. Instantaneously he found himself, still holding the Liberator, in London Zoo, near the aquarium. "My God, the thing actually works, spatially at least", he exclaimed to himself. "Let's see if it has delivered temporally." Spotting a newspaper kiosk, he marched over to it and bought a copy of the *Times*. He was thrilled to find the newspaper's date to be 28 May 1910, precisely 10 days from the time he had pushed the *Execute* button. He exulted in the realization that he had become a time -traveller, a *genuine chrononaut!* Seeing another button on the Liberator labelled *Return to origin,* he pushed that and instantly found himself back in his study, where his copy of the Times of 18 May 1910 still rested on his desk. He noted that he had actually been returned to a moment slightly ahead of his time of departure, in order, presumably, to prevent the possibility of paradoxes of the kind described in the Time Traveller's letter arising on his return. So apparently some "anti-paradox" circuitry had been built into the device after all.

It suddenly struck him that this, his very first use of the Liberator, had already placed him in a position in which he could generate one of the paradoxes described in the Time Traveller's letter. For if he waited 10 days and walked to the aquarium at London Zoo he would almost certainly meet his younger self on the latter's arrival 10 days in the future. This sort of situation the Time Traveller and his colleagues had vowed to avoid, and despite a curiosity to meet his younger self which had suddenly arisen in him, he resolved to steer clear of the Zoo entirely for the next couple of weeks.

The prospect of travelling the world opened up by the Liberator had brought him a fierce delight, along with a boundless feeling of gratitude for the Time Traveller's generosity. But this was accompanied by a certain frustration at not being able to tank his benefactor since the letter did not contain a return address. *Faute de mieux,* he uncorked a bottle of champagne from his cellar, and, pouring and raising his glass, declared, "I salute _______, genius and benefactor, and I vow to do so every 18th of May."

Now he had to consider to what purpose he should devote this amazing device he had been so miraculously bequeathed. He reflected that he had always admired the talent and creativity of exceptional persons, in every realm of human endeavour. Why should he not employ the Liberator to take him to such places and times as to make it possible to meet and talk with these individuals? He could see that in practice these conversations would necessarily be limited to those who spoke the languages in which he was fluent: English, French and German. And the times at which these conversations could take place would also have to be restricted. For example, he would be delighted to discuss Shakespeare's plays with their creator (whoever he was), but the four centuries separating Bennhall's version of English from the Bard's would almost certainly make their speech mutually incomprehensible. These constraints meant that his range of operations could not extend further back in time than the 17th century, and would have been confined to the Western Hemisphere. Still, he thought, a rich field to mine.

Achieving his ambitions, he realized, would require considerable sums of money, which, having drawn heavily on his inheritance, he no longer possessed. But was it not he who had originally observed that, if one could travel into the future, a profit could be made by investing one's money at interest and "hurrying on ahead"?

Yes, of course, and the Time Traveller himself had acted on his observation, accumulating the hundreds of millions of pounds required for the development of the Liberator. Bennhall's own purposes would not demand anything like a sum so vast, several million pounds would certainly do. Accordingly, the following day he went to his bank and invested all but £500 of his inheritance in fifty-year bearer bonds, at a high rate of compound interest. He arranged to have the bonds stored in a safety deposit box and for the bank to attach the associated interest coupons as they were issued. He had the remaining £500 converted into miniature one ounce gold bars. His intention was to travel 50 years into the future and cash in the bonds, which by that time would have increased greatly in value. It had crossed his mind that doubts might be raised if he, a 40-yeaf-old man , showed up at the bank to cash in bonds which had ostensibly been purchased 50 years earlier. He saw that, to allay suspicion, he would have to call in at the bank disguised as an elderly man. In any case, the ability to disguise himself would likely prove useful in his prospective adventures in time. He located a theatrical costumier in the Strand who was able to supply what he needed, make-up, wig, pince-nez and all. On the morning of his departure, he applied his camouflage and inspecting his appearance in the mirror, winked at himself and chuckled "Who needs Sherlock Holmes?" with satisfaction. He put his stash of gold bars in a capacious money belt he had purchased for the purpose and tightened it around his waist. Taking up the Liberator, he set it to take him 50 years in the future to an alley off Trafalgar Square which he knew was normally deserted. The image which appeared on the Liberator confirmed that this would still be the case in 50 years time. Pushing the *Execute* button, he was instantly deposited in the alley, arriving, as he had planned, unseen. Walking into Trafalgar Square, he was astonished by the buzzing confusion of the traffic, a stream of self-propelled, rubber-wheeled steered metal vehicles conveying their occupants at high speed. He soon came across a bank, where he was able to convert his gold bars into the current funds without difficulty, and was surprised to receive in excess of £1500, which he supposed was the result of inflation in the intervening half century. He next proceeded to a nearby hotel, where he rented a room for a week. Finally he directed his steps to his own bank, whose name, he found to his concern, had undergone a change since he had last entered it. But this turned out to be of no consequence, for on entering and asking for his safety deposit box he was merely required to verify his signature. He was then conducted to the vault, where he was presented with the box, which had rested there for half a century. Unlocking it with the key originally issued by the bank, he opened it and extracted the bonds, which he was pleased to see had all the accumulated interest coupons attached. He returned the box to its receptacle and asked to see the bank manager of the bank. In the manager's office, he was invited to sit down and was offered a cigar, which he accepted and lit up. The civilities concluded, he got down to business.

"I'd like to cash in some bearer bonds," he said.

"Certainly," replied the manager. "May I see them, please?"

He handed the bonds to the manager, who scrutinized them for a time and then said, in a tone of astonishment,

"My dear sir, do you realize that with the accumulated interest from the attached coupons, these bonds, which I see were issued in 1910, are now worth in excess of four million pounds?"

"Yes, that does not surprise me. I calculated that they would eventually become worth that much when I bought them fifty years ago."

"Well, I must congratulate you on your sagacity, and even more on your patience in waiting half a century for these bonds to mature."

"Thank you, sir."

"How would you like to receive your funds?"

"In one ounce gold bars."

'"That is an unusual form of disbursement these days, but it can be arranged. Are you aware that at today's gold price, four million pounds sterling's worth of gold would come to more than two hundred pounds in weight?"

"Yes, I had anticipated that. I'd be obliged if you could have the gold delivered to my hotel, which is not far from here."

"Of course. If you'll just give me the name and address of your hotel, I'll arrange the delivery right away."

"Thank you, it's been a pleasure doing business here."

The following day the gold was delivered to Bennhall's hotel. He was now faced with the task of transporting 200 pounds in weight of gold bars to his Regent's Park house in 1910. He achieved this by making a succession of trips back and forth in time, carrying on each trip 20 pounds of the bars in his money belt. To avoid meeting himself on these temporal oscillations he was careful to time each return trip to occur a little later than the previous departure. After his final trip, he deposited the accumulated gold bars in a safe he had installed for this purpose. The Regent's Park house in 1910 was to serve as his base of operations throughout his travels in time.

If he were to travel about and spend extended periods of time in the 20th century, he foresaw that he would need the appropriate documentation for identification purposes. He guessed that this documentation would take the form of a passport of some kind. Accordingly he used the Liberator to travel forward to London in the year 2000 to ascertain what form such a passport would take, and how to obtain it. He located a public library with a good reference section where he consulted a British government document outlining the regulations for obtaining a passport. He found that during the period between the introduction of the standard 10-year British passport in 1920 and the year 1983 , to obtain a passport one needed only to submit a British birth certificate. After 1983 a change of regulations made it mandatory also to submit the British birth certificate of one parent. Simplest, then, to confine his efforts to the years before 1983. British passports being valid for 10 years, he would need to obtain passports valid for each decade between

1923 and 1993. He returned to 1910 to ponder the problem, and suddenly hit on the answer. It was simplicity itself. He had merely to travel via the Liberator to the first day of each decade and tour graveyards looking for the headstone of a boy who had been born roughly 40 years before the beginning of the decade and who had died before the age of, say, 10. He would then take the deceased boy's details to the Central Registry of Births, Marriages and Deaths, and purchase a copy of the boy's birth certificate - all the proof needed to apply successfully for a passport. He was later to find that this devious method of obtaining a passport was, oddly, not blocked by the British authorities until 2007.

To carry out this ingenious plan he made a total of 7 trips into the future, again taking care to time each return for a little later than the previous departure. He now had British passports valid for the years 1923 -93.

The necessary arrangements that would make the world his oyster, both in space and time, were now complete. He felt as if he possessed the boundless powers of an Ariel.

But he had not as yet used the Liberator to travel any substantial distance in *space*. It happened that he had read an article in the *Encyclopedia Britannica* on the Forbidden City in Peking in China and was curious to see it. Why not, he thought, pay it a visit a century or so after it had been built during the Ming dynasty? Picking up the Liberator, he selected a date in the late 16th century, then pushed its various buttons until its screen displayed the flashing sign *(b) other location.* He pressed his fingertip on this and again the screen displayed a map of London, superimposed by a grid, accompanied by a small movable arrow. Now he noticed another button on the device marked *Move origin of coordinates.* He pressed this and immediately the grid began to flash. He found that if he placed the arrow on the grid and tried moving it in a certain direction, both arrow and grid remained still but the region displayed by the map moved in that same direction. If he attempted to move the arrow to right or left, the area displayed by the map moved eastward or westward; if the arrow was urged upward or downward, then the map moved northward or southward. In this way he succeeded in manipulating the map so that it to displayed the region surrounding Peking. Moving the arrow to a square over central Peking and pushing the *Expand view* button, he was rewarded with a view of the Forbidden City. He pushed the *Execute* button and instantly found himself in what he took from the description in the *Encyclopedia Britannica,* to be its Imperial Garden. It seemed deserted, and his sudden materialization there went unobserved. This afforded him the opportunity of touring a unique site which, at the time of his visit no Westerner (and few Chinese) had ever seen. The Garden's *Britannica* description had called it an exquisite place and he saw it was all of that: the delicate harmony of its trees, rock gardens, flower beds and bronze incense burners reminded him of Coleridge's *Xanadu*. Among its buildings he noted the four pavilions situated at the garden's four corners which he knew symbolized the four seasons. He was just about to inspect the Spring Pavilion when a small group of people suddenly emerged from the building, and spotted him immediately. His white face and Western garb identified him as an evident interloper, and they quickly raised the alarm. He took to his heels, and, when he could seize the opportunity, pushed the *Return to origin* button on the Liberator. He was instantly returned to his study in London. Despite his undignified exit from the Imperial Garden, he was mightily pleased with his adventure. He had mastered the use of the Liberator as a spacetime vehicle, and in doing so had visited a legendary place at its apogee.

Chapter 3

Bennhall Surveys the 20th Century

Bennhall was now ready to embark on his travels in time and space, his avowed purpose the search for persons of talent and creativity. Now of course, what knowledge of such individuals he possessed was confined to his own past, that is, to the years before 1910. His encounters during the previous decade with intellectuals, artists and musicians in Paris and Vienna all of whom, with rare exceptions, were excited by the prospect of the future, had led him to the belief that the coming century would prove to be one of remarkable achievement. He had the sudden realization that, while he admired the glories of the past, he was equally excited by the future, sufficiently excited, in fact, to focus his search for creative talent to the coming century and beyond. If he ever tired of mining the talent of the future, he could always return to the past to seek out his favourite achievers there.

How should he set about identifying those future luminaries? After pondering the matter for a while, he decided that the best way of doing this was to locate a good library in London late in the 20thn century and consult its reference section. There were many such libraries in London, the most prominent being the British Museum Library and the London Library in St. James' Square. Accordingly he placed a generous supply of gold bars in his money belt, strapped it on, packed a small bag containing his 1983 passport and set the Liberator to take him to Hyde Park early on a winter morning in the year 1990, figuring that at that time there would be few people about to witness his precipitate arrival there. He was right in this surmise and arrived unobserved. He sought out a bank and cashed some of the gold bars into current pounds, at the same time asking the teller if he could recommend a good hotel in the vicinity. The teller suggested a hotel just a few streets distant, to which he made his way. The hotel looked luxurious and well equipped, with what appeared to be an excellent restaurant, so he had no hesitation in registering there, taking a room for a month. The following day he became a member of both the British and London Libraries, offering his 1990 passport as identification. He spent the ensuing month delving into the resources of their reference sections. In his search for outstanding creative and accomplished people of the 20th century, he perused encyclopedias, dictionaries and histories of 20 century art, music and literature, histories of 20th century philosophy, science and mathematics. Particularly useful in the arts and humanities were the *Companions* which had been published by the Oxford University Press since the 1930s. By the year 1990, this ambitious series of volumes had expanded to embrace a vast range of topics in the humanities, providing him with much pertinent information about the development of the arts in the 20th century. He was intrigued to find that the first volume in the series, *The Oxford Companion to Literature* of 1932, had been edited by a distant relative of his whom he had known in his youth and who died in 1948.

As he pursued his researches into the culture of the 20th century, and began to identify the persons who had made outstanding contributions to it, he came to realize that he would need to devote an extended time to the study of their achievements He would read their novels, listen to their music, analyze their scores, see and hear their performances, view their paintings, study their philosophy, grasp their theorems, understand their theories. He resolved to devote a year to this intense, but for him pleasurable activity. In addition to the information obtainable from the printed word and musical recordings, he would attend concerts, go to the cinema and the theatre, and frequent art galleries.

This meant that he would have to seek out more settled accommodation. Accordingly he visited a local estate agent and arranged to rent am apartment in the vicinity for a year .

The year he spent in late 20th century London was both instructive and exciting. In his researches he acquired an extensive knowledge of the turbulent history of the 20th century, which had only been briefly sketched in the Time Traveller's letter. He learned of the First World War of 1914-18, in which 20 million died; the Russian Revolution of 1917, the resulting installation of Communism and the transformation of Russia into the Soviet Union under Lenin, and later Stalin; the rise of Italian Fascism with Mussolini in 1922; the Japanese invasion of northern China and the installation of a puppet state there in 1931; the rise of German Nazism with Hitler in 1933; Hitler's invasion of Poland in 1939 initiating the Second World War of 1929-45 with its 75 million deaths and huge displacements of populations; the murder during that war by the Nazis of 6 million Jews, many in extermination camps expressly designed for the purpose; the development by the United States of the atomic bomb which it used on Japan in 1945, killing 200,000 people; the final victory in 1945 of the United States, Great Britain and the Soviet Union over Germany, Italy and Japan; the establishment of the United Nations at the end of that same year; of the victory in 1950, after a long civil war, of the Chinese Communist armies and the subsequent transformation of China into a communist state under Mao Tse-Tung; the Cold War and the arms race between the United States and the Soviet Union which had ongoing since 1947. (Bennhall only learned later of the collapse of the Soviet Union at the end of 1991.)

He gained a detailed knowledge of the extraordinary developments in science and mathematics in the 20th century, a few of which had been mentioned by the Time Traveller. He learned of the formulation of the general theory of relativity by Einstein in 1916, in which gravitation is identified with space-time curvature and Newtonian cosmology based on Euclidean geometry is replaced by a radically new account of the structure of the universe based on non-Euclidean, Riemannian geometry; the discovery in the 1920s of the universe's expansion and the formulation of the " Big Bang" theory of is origin; the rise in the 1920s of quantum mechanics to reign over the world of atoms and subatomic particles, a realm in which the solid intuitions of classical physics, such as continuity and determinism, are replaced by discreteness and chance; the development in the middle of the century of particle physics and its exotic zoo of elementary particles serving as the building blocks of the material world.

In mathematics he learned of the rise of set theory, created by Cantor in the late 19th century, to a position of dominance in the foundations of mathematics; the emergence of abstract algebra, and, in the 1940s, of the theory of categories as a unifying scheme for the whole of mathematics; the development of the subject of topology as a vast generalization of geometry; the later emergence of sheaf and topos theory unifying algebra, geometry and logic; the rapid development, in the 1920s and 30s, of mathematical logic and the proof by Kurt Gödel in 1931 of the Incompleteness Theorems asserting that it will always be possible to formulate mathematical propositions which can be neither proved nor refuted, and that the consistency of mathematics can never be rigorously proved.

He was awed by the dazzling efflorescence of the arts and humanities in the 20th century. As a former pupil of Schoenberg, he was acquainted with the modernist atonalism developed by the master and his disciples in the first decade of the century. He learned of the impact, and controversy, of the first performance in Paris in 1913 of Stravinsky's *Rite of Spring*, and was amused by its description in the French press as "a laborious, puerile barbarity" and later comments such as "he who could write *The Rite of Spring*, if I be right, by right should swing". He learned of the invention of the serial method of composition by Schoenberg in the 1920s and its pervasive influence on later composers, but which never became popular with audiences; Stravinsky's contemporaneous development of neoclassicism, for which Schoenberg showed such scorn; the initially expressionist, and later neoclassical compositions of the amazingly prolific German composer Paul Hindemith; the radically modernist yet folk influenced music of the Hungarian composer Bela Bartok; the impassioned works of the Swiss-born composer Ernest Bloch (Bennhall was amused to learn that all five of these composers had become American citizens). He also became acquainted with the eclectic, original and often tuneful compositions of the Russian composer Serge Prokofiev, who was to become perhaps the most popular classical composer of the century; the few but elegant works of the French composer Maurice Ravel; the haunting music of the American composer Aaron Copland; and the bubbling, exotic music of the Brazilian composer Heitor Villa-Lobos. He was instantly taken with jazz, the improvisational musical genre invented by Black Americans in the first decades of the century, which had evolved by the middle of the century into a sophisticated, rhythmically and harmonically subtle form of music - truly the improvisational face of modernism. He also came to enjoy the popular rock music of the later 1960s, particularly that of the British group The Beatles.

His intensive study of 20th century literature began with James Joyce's *Ulysses* of 1922, widely regarded as one of the two greatest works, along with Marcel Proust's colossal novel *A la Recherche du Temps Perdu,* of 20th century modernist literature. Joyce's account of Leopold Bloom's one day Odyssey through Dublin on 16 June 1904 did not resemble, even remotely, any novel he had read. He was deeply impressed with its originality and ingenuity of construction . Joyce's use of a "stream of consciousness" technique to enter the actual thinking of his characters h considered particularly inspired. He was astonished by Joyce's erudition and the sheer range of his vocabulary, which continually transcended the bounds of the best dictionaries. His final assessment of *Ulysses* was that it could not be considered a novel in the sense that he understood the term. But it was undoubtedly a masterpiece.

By contrast, he had formed a mixed impression of Marcel Proust's *A la Recherche du Temps Perdu.* Before embarking on his travels in time he had not then heard of Proust's novel, one of the longest ever written, since it was published in several volumes, the first of which appeared in 1913. After its brilliant beginning with the *Overture,* he found the novel, with its interminably long sentences, somewhat heavy going, and he had to agree with one critic's caustic observation:

Everybody tries to climb Mt. Proust, though many a stiff body is found on the lower slopes, with the other readers stepping over it gingerly.

In beginning his investigation of the 20th century novel with *Ulysses,* Bennhall had taken the modernist movement in literature by the horns. He followed up with works which, while less demanding on the reader, made their own claims to originality: Thomas Mann's *The Magic Mountain* and *Dr. Faustus;* Robert Musil's *The Man Without Qualities;* Hermann Broch's *The Sleepwalkers* and *The Death of Virgil;* Hermann Hesse's *Steppenwolf* and *Magister Ludi;* Elias Canetti's *Auto-da-Fe;* Franz Kafka's *The Trial, The Castle* and his remarkable short stories; Evgeny Zamyatin's *We*; Vladimir Nabokov's *Lolita, Pnin,* and his autobiography *Speak, Memory;* Jean-Paul Sartre's *Nausea* and *Roads to Freedom;* Albert Camus' *L'etranger*; George Perec's *Life, A User's Manual*; William Faulkner's *The Sound and the Fury* and *Light in August;* Jorge Luis Borges' *Ficciones*; Gabriel Garcia Marquez's *One Hundred Years of Solitude;* Aldous Huxley's *Point Counter Point* and *Brave New World;* George Orwell's *1984,* Dashiell Hammett's *The Maltese Falcon* and Raymond Chandler's *Farewell My Lovely.* He also became captivated by *science fiction*, the literary genre which had been initiated by H.G. Wells at the end of the 19th century, and which had blossomed in the 20th. The novels and short stories of Karel Čapek Olaf Stapledon , Philip K. Dick , Stanislaw Lem, J.G. Ballard , Alfred Bester, and William Tenn stood out in his estimation.

He was impressed with the poetry of the 20th century, in which the romantic traditions of the early 19th century had been almost entirely abandoned. As with the other arts, poetry in the new century was infused with a spirit of experimentation and breaking of boundaries, following on from the French Symbolist poets of the later 19th century. Among 20th century poets he took an especial liking to the work of T.S. Eliot, W.H. Auden, Louis Macneice, Paul Eluard, Rainer Maria Rilke, Eugenio Montale, Alexander Blok, Dylan Thomas, Theodore Roethke, Sylvia Plath, Philip Larkin, William Carlos Williams, e.e. cummings, Constantin Cavafy; and the later work of his near-contemporary W. B. Yeats.

He was astonished by the enormous strides made in the 20th century by the performance and recording of music. He noted the emergence of a perfection of technique on every instrument that would have been unimaginable in previous eras - in his eyes, by the later 20th century, every professional musician had acquired the technique of a virtuoso. This was also true of orchestras and their conductors; the orchestra had itself become a composite virtuoso conducted by a virtuoso of the baton. He saw that this extraordinary rise in technical achievement had three essential causes: the influence of individual virtuosos with transcendent techniques, starting with the nineteenth century violin virtuoso Niccolo Paganini; the expansion of professional music schools adhering to the highest standards; and the rapid development of recording and communication technology.

He was passionately devoted to the violin and had heard many of the greatest violinists of his day perform, including Sarasate, Ysaye, and Kreisler. In November 1910, following his return to London, he heard Kreisler give the first performance of Elgar's violin concerto at the Queen's Hall, with the composer conducting. He had been deeply moved by Kreisler's performance, and was delighted to learn that Kreisler had lived to the ripe old age of 86, dying in 1962.

In the course of his investigations, he soon identified the 20th century violinist who could be considered equivalent to Paganini in terms of technique and influence. This was the Russian-American violinist Jascha Heifetz, born in 1901. Heifetz's spectacular violin playing, with its uncanny accuracy, drive and purity of intonation, quickly established itself as the standard for professional violinists to strive for, even if very few succeeded even in getting close. Heifetz maintained his near-superhuman technique though a rigorous regime of practicing, and was quoted as saying, "If I don't practice one day, I know it; two days, the critics know it; three days, the public knows it." Bennhall was delighted to learn of Fritz Kreisler's remark, after the young Heifetz's debut in Berlin in 1912, "Well, gentlemen, we may as well break our fiddles across our knees.'" Bennhall was also highly amused by the famous anecdote, long circulated by 20th century violinists, concerning Heifetz's debut recital at Carnegie Hall in New York in 1917. It was reported that Mischa Elman, a famous violinist of the day, who was in the audience, at one point during the recital turned to the man in the next seat, who happened to be the equally famous pianist Leopold Godowsky, and remarked, "It's hot in here, isn't it?". Godowsky's reply, "Yes, but not for pianists" became the stuff of legend. Bennhall resolved to be present at that recital himself. The thought quickly followed of travelling to the early 19th century to witness Paganini perform. In that way Heifetz's and Paganini's playing could be directly compared.

Heifetz became one of the greatest recording artists, making his first official recordings in 1917 following his New York debut. It was through listening to these many incomparable recordings that Bennhall first became acquainted with Heifetz's playing. He came to regard Heifetz's 1952 recordings of the Bach solo sonatas and partitas as the greatest musical performances of all time.

With Heifetz supreme among them, the 20th century blazed with great violinists: Yehudi Menuhin, David Oisrrakh, Nathan Milstein, Ruggiero Ricci, Isaac Stern to name a few. Bennhall listened to their recordings with relish, but he was determined to see and hear every one of them in the flesh.

He was also a devotee of the cello, and had heard the great Spanish cellist Pablo Casals (just a few years younger than he was) play in Paris in 1899. He learned that Casals had continued to be active, and highly influential, well into the 20th century, dying in 1973 at the age of 96. He was bowled over by Casals' 1936 recordings of Bach's cello suites, and accorded them a veneration comparable to Heifetz's Bach recordings.

He was delighted to see how the cello had flowered as a solo instrument in the 20th century, its popularity with listeners rivalling that of the violin. At the same time, as with the violin, technical standards of cello playing rose steadily. He was overwhelmed by his first hearing of the sonata for solo cello composed in 1915 by the Hungarian Zoltan Kodaly. This, the first work for solo cello to be composed since Bach's time, is a technical *tour-de-force* which played no small role in raising the standards of 20th century cello playing. He noted the unprecedented abundance of brilliant 20th century cellists (in addition to Casals): Emanuel Feuermann, Gregor Piatigorsky, Paul Tortelier, Pierre Fournier, Mstislav Rostropovich, Leonard Rose, Janos Starker, Aldo Parisot, Jacqueline du Pre, Yo-Yo Ma… He was convinced that cellists would be entirely justified in claiming the 20th century as their own.

The 19th century had seen the emergence of the piano as a virtuoso instrument, and the musical scene soon became crowded with brilliant, flamboyant pianists bent on dazzling the public with their prowess at the keyboard. Himself an accomplished pianist, Bennhall had witnessed the later emergence, among younger virtuoso pianists - such as his contemporaries Ferruccio Busoni, Leopold Godowsky, and Josef Hoffman - of higher standards, both in technical precision and self-discipline. He was pleased to observe that in the 20th century this tendency had continued to spread. In the 1920s the playing of the Russian pianist Vladimir Horowitz offered a sensational fusion of expression and technical precision which emulated on the piano what Heifetz had achieved on the violin. Bennhall was deeply impressed by the Austrian pianist Artur Schnabel, whose dedicated, focussed performances of the piano compositions of Mozart, Beethoven and Schubert he felt epitomized the classical tradition. Of the many brilliant 20th century pianists, there were four who stood out for Bennhall above the rest, the Russians Vladimir Horowitz and Sviatoslav Richter, the Italian Maurizio Pollini, and the Canadian Glenn Gould. Of these remarkable musicians, Glenn Gould struck Bennhall as the most eccentric and original. Gould was best known for his Bach playing, but he was also a passionate advocate of contemporary music, especially that of Schoenberg and the Second Viennese School. Although he lived to the age of fifty, his career as a concert pianist was very short, amounting to just 8 years. Bennhall became fascinated by Gould's "antivirtuoso"attitude to public performance, famously declaring "at concerts I feel demeaned, like a vaudevillian". Gould declared that live concert performances were an anachronistic practice, and that the future of music lay in recording and electronic media such as television. The purpose of art," he wrote, "is not the release of a momentary ejection of adrenalin but is, rather, the gradual, lifelong construction of a state of wonder and serenity." In 1964, Gould left the concert stage at the summit of his performing career to devote himself to recording, broadcasting, composing and writing. Renarkably, , his fame amongst the general public actually increased.

Bennhall saw how the rapid development of recording and broadcasting technology in the 20th century came to revolutionize the reproduction and diffusion of music. The steady technological progress from the primitive acoustic recordings of the early decades of the century, through the 78 rpm shellac discs of the middle decades, to the vastly superior long playing vinyl records of the 1950s and 60s, and thence to the tape cassettes and compact discs of later decades — these developments made possible the storage and perfect reproduction of musical performances and the universal access thereto. The parallel development of radio, and, in the 1950s, television, enabled live performances to be brought to every home.

Bennhall was struck by the diversity of 20th century art movements, as reflected in its profusion of "isms": Fauvism, Cubism, Orphism, Purism, Futurism, Expressionism, Suprematism, Constructivism, Vorticism, Dadaism, Surrealism, Abstractionism, Precisionism, Social Realism, Socialist Realism, Abstract Expressionism, Neo-Expressionism, Photorealism, Conceptualism. Of course, these "schools" were by no means mutually exclusive, and there were a number of eclectic artists whose work manifested features associated with several "schools".

As a mathematician, Bennhall was fascinated to find that contemporary developments in mathematics had exerted an influence on the work of a number of 20th century artists. This was especially true of the members of the Cubist school which emerged in the first decade. At that time certain artists began to abandon the traditional technique of perspective. These artists – the Cubist painters led by Georges Braque and Pablo Picasso

– introduced a new technique which, instead of attempting to produce the impression of three dimensions, brought the bidimensionality of the canvas into the foreground. To achieve this they decomposed objects into elementary geometric forms such as planes, cubes and pyramids which were then reassembled and presented in a relieflike way on a flat or shallow space. This technique dominates Picasso's early cubist paintings, in particular the famous *Les Demoiselles d'Avignon* (1907), with its radical distortion of figures and presentation as fractured planes. In 1908 Braque introduced geometric cubism, in which natural objects such as trees and mountains were presented as shaded cubes and pyramids. Bennhall was amused by the contemporary art critic Louis Vauxcelles piqued reaction in describing Braques' representations as *bizarreries cubiques* , which apparently gave the movement its name. He noted that a number of contemporary critics thought that in abandoning perspective the cubist painters were taking a step backwards. Vauxcelles had referred to the members of the cubist movement as *ignorant geometers, reducing the human body, the site, to pallid cubes*. But Bennhall came to see that from a mathematical standpoint, the cubists were actually making an advance. For he had noticed that their technique of decomposing objects into elementary geometric forms is closely analogous to the central idea of combinatorial topology, a branch of mathematics which was emerging at about the same time, led by the great French mathematician Henri Poincaré. Here the central idea is the investigation of the properties of topological spaces by subjecting them to combinatorial decomposition into simpler spaces such as simplicial complexes, constructed by "gluing together" points, line segments, triangles, and their higher dimensional counterparts. The planes, cubes and pyramids of the Cubist artists correspond to the simplicial complexes of the mathematicians. Bennhall had also noted that Picasso, along with a number of his fellow cubists, had read Poincaré's important work *Science and Hypothesis*. They had been deeply impressed by Poincaré's claim that Euclidean geometry, as applied to space, was essentially a convention, rather than an absolute truth, and that other, noneuclidean geometries might be equally acceptable as descriptions of the structure of space.

Bennhall observed the influence that mathematics had exerted on other 20th century artists. The De Stijl movement, founded in 1917 by the Dutch artists Theo van Doesburg and Piet Mondrian had the aim of aim of establishing "a visual vocabulary composed of elementary geometrical forms comprehensible by all and adaptable to any discipline". The Dadaist artist Man Ray photographed some of the mathematical models in the Institut Henri Poincaré in Paris, including *Objet Mathematique* (Mathematical object). The great surrealist artist Max Ernst painted Lissajous figures, the mathematical curves representing complex harmonic motion, by swinging a punctured bucket of paint over a canvas. Bennhall was also delighted by the work of the mid-century Dutch graphic artist M.C. Escher, which is animated by the use of tesselations, the geometry of polyhedra, topology, and self-reference. Towards the end of his life the great Spanish surrealist Salvador Dali painted a series of pictures, including *Topological Abduction of Europe and The Swallow`s Tail,* inspired by Catastrophe Theory, a mathematical method for describing the evolution of natural forms created by the French mathematician René Thom.

Bennhall was intrigued to find that cognitive paradoxes had also stimulated the imaginations of certain 20th century artists, particularly that of the Belgian surrealist René Magritte. One of his favourite examples of this was Magritte's painting *La Condition Humaine* of 1933, which features an easel on which a painting sits smoothly blending into a view through a window framed by "actual" curtains in the painting. Anotther was Magritte's 1942 work *The Domain of Arnheim,* which depicts a shattered window whose shards lying on the floor still show the outside world they originally concealed. He noted Escher's indulgence in similar conceits, for example *Print Gallery,* which is itself a print presenting a distorted city containing a gallery which contains the print, and so ad infinitum.

Bennhall also conceived a passion for the work of the Fauvist painters Matisse and Derain, the cubists Braque and Picasso, the surrealists Miro, Dali, Tanguy and Magritte, the Dadaist Max Ernst, the abstractionists Kandinsky, Klee, Delaunay, Arp, Mondrian and Appel, the abstract expressionist Jackson Pollock. Of the artnworks of the 20th century, he came to revere above all the 23 paintings in Miro's *Constellation* series of the 1940s. These masterpieces of abstract art, each small in dimensions but possessing a subtle intricacy, moved him in a way that he found difficult to describe.

The beginnings of what was to become the major new art form of the 20th century, the motion picture, lay in the later 19th century. In the 1890s Bennhall had seen some of the films of the pioneer French director Georges Melies , and had found them amusing but somewhat crude. He was astonished when he saw the later advances, made primarily in the United States, in the production of motion pictures, transforming what had seemed to him primitive into an art form both technically sophisticated and universally popular. The "movies", as they came to be called, turned actors, the "stars", into modern gods and goddesses. Such was the technical accomplishment of the moviemakers that audiences did not notice the extraordinary collective effort which went into the making of a film:

the director, producer, scriptwriter, camera operator, musical score composer, film editor, special effects creator, makeup artist, and numerous others, all playing essential roles in fashioning the seamless result the audience finally viewed on the screen. He saw the motion picture as the ultimate cooperative art form, far transcending the theatre as a collective enterprise.

The theatre itself survived nevertheless, if on a reduced scale, largely because of the continuing impact appeal to audiences of live performances by actors. This was analogous to the survival of live musical concerts, owing to the continuing appeal of live performances by musicians, especially in the popular field. Bennhall was intrigued when he learned that the movie "industry", centered from the beginning in Hollywood, California, underwent in the 1950s its own struggle with the emerging rival medium of television, centred in New York City.

Moviemaking was a collective enterprise, but as far as the public was concerned there was only one category of movie personnel that really mattered- the actors, the "stars". Like everybody else, Bennhall too fell under their spell, from the stars of early silent movies such as Charlie Chaplin, Buster Keaton and Charles Lloyd, through those of the "talkies" of the 1930s, Greta Garbo, Joan Crawford, Bette Davis, Fred Astaire, Edward G. Robinson, Fredric March, James Cagney, Spencer Tracy, Cary Grant,…; the 40s, Katharine Hepburn, Humphrey Bogart, James Stewart, Orson Welles, Laurence Olivier, George Sanders, Robert Mitchum, …; the 50s, , Kirk Douglas, Burt Lancaster, Marlon Brando, Gene Kelly, Elizabeth Taylor, Ricard Burton…

As he had anticipated, Bennhall's researches into the 20th century yielded a constellation of creative personalities. He now began to think about how to arrange to meet them. One thing that stood out was the fact that by 1990, the year he had begun his researches, many of his subjects had already died, and in most cases he knew the time and manner of their deaths. It struck him that this knowledge made him a latter-day Lachesis, the ancient Greek Fate who measured the thread of each person's life. Bennhall, like Lachresis, had no intention of revealing this knowledge to his subjects, nor did he (at least at first) want to give them the slightest inkling that he was a chrononaut. But he was uncertain as to what his emotional reaction might be when actually faced with someone whose time of death he knew in advance. He thought long and hard about this, finally deciding that if he became too upset at his first such meeting, he would not proceed with the project.

How exactly to present himself at the face-to-face meetings he had in mind also required some thought. He eventually came up with the mildly deceptive idea of introducing himself as a representative of a cultural publication or magazine and requesting an interview. Of course, this would amount to a double deception since the conversation (as he chose to regard it) would never be published, but he felt that no real harm would be done. The plain truth was that he could not see any other way of achieving his goal. He realized that he would have to supply an alias and have cards printed up with that name presenting him as an agent of the appropriate magazine. He fancied the idea of using an anagram of his own name as an alias, but because of its surfeit of 'L's could only produce the somewhat contrived names "John Lane Bell" , "John Lane-Bell" and "John Bell-Lane. He had settled on the last of these, but then realized that the contraction "Bellane" sounded much more plausible. And it appealed to him as being the name of an excellent Rhone wine - Côte Bellane - which he had discovered on one of his exploratory trips into the future. So "John Bellane" was launched into the world.

He was fully aware that in making these numerous trips in time he ran the risk of inadvertently generating paradoxes and duplicating universes. He knew that he had to proceed with great caution. For instance, he felt it wise to visit each subject no more than once. This would eliminate the possibility of visiting a subject for the first time in both his and the subject's lives, and then subsequently (in his own lifeline) returning to visit the same subject at a time (in the subject's lifeline) *before* that first time. This would generate a paradox because the "first" time the subject could have no recollection of a former visit by him, whereas, once the second visit took place, the "first" time he appeared, the subject would recall that former visit, and presumably mention it to him. Thus, from his point of view, an inconsistency would surface: by visiting the subject the second time, the subject's behaviour on the first visit would deviate from what he himself actually remembered.

He resolved not to allow bizarreries such as these to disrupt his travels.

It occurred to him that he needed to obtain a discreet recording device in order to preserve the conversations with his subjects. From what the Time Traveller had said in his letter, it was clear that by the year 2020 miniaturization of electronic devices would have been perfected, Accordingly he used the Liberator to travel to that year and purchase a miniature recording device. He found one at a store in London which, being only 2 inches long, could easily be concealed in his pocket. It also had many thousands of hours of recording capacity, so was perfect for his purposes. He called it his "Total Recall".

He started a journal in which to set down and enlarge upon his transtemporal encounters.

Part II

Bennhall's Transteomporal Journal

Heifetz and Paganini

The object of my first serious transtemporal journey was to attend Jascha Heifetz's legendary debut concert in New York's Carnegie Hall, which I knew was to take place on 27 October 1917. In order to secure a good seat in the hall, I time-travelled to New York City so as to arrive there some days in advance of the recital. I was able to obtain a seat in the centre of the third row.

The evening of Heifetz's recital was unseasonably warm, and the hall was crammed to suffocation. I was glad that I had chosen to wear just a light linen suit for the occasion. As I took my seat, I switched on my pocket recorder, and looked around to see if I could spot Elman and Godowsky. Yes, there they were, sitting side-by-side in the second row just a few seats along. I wondered, not for the first time, whether the future is predetermined. But this thought was quickly dispelled by the entry onto the stage of Heifetz and his accompanist. Heifetz was just 16 years old at this time, but his poise and dignified bearing as he made his way onto the stage made him look considerably older. After a brief bow to the audience, he took up his violin, raised his bow and launched into the Vitali *Chaconne.* The effect was electrifying, even to me, who, uniquely among the audience, was familiar with the mature Heifetz's recording of the work 40 years in the future. The fleckless purity, the seemingly effortless perfection of the young Heifetz's playing was truly astonishing. Now I truly understood the impact this young man's playing had on the professional violinists present on the occasion. His mastery of the instrument, at such a tender age, must indeed be striking them as unprecedented. This lent considerable plausibility to the famous exchange between Elman and Godowsky that was supposed to have taken place that evening. I noticed Elman occasionally mopping his brow with his handkerchief and exchanging a few words with Godowsky between numbers on the program, but I could not make out the words. After the interval, Heifetz continued his mesmerizing performance, concluding with the arrangement of Paganini's 24th Caprice by his own teacher Auer. At the finish the audience rose to its feet and erupted into applause, with numerous shouts of "Bravo" and "Encore". This went on until Heifetz modestly granted the audience's wishes and delivered several encores. After Heifetz left the stage, I joined the rest of the audience slowly making its way out of the hall, hand once again resolved to make a direct comparison of Heifetz's playing with Paganini's.

This ambition required that I actually attend a Paganini performance. I decided to time-travel to Paganini's final London concert, which took place at the Drury Lane theatre on 8 August 1833. As before, I aimed to arrive a few days in advance of the concert so as to obtain a good seat for the performance. I took a room at Mivart's hotel in Mayfair, one of the few hotels in London in that period, a modest establishment which a couple of decades later was to evolve into the famous Claridges.

The London of the 1830s dis not appear markedly different from the London of 1910. The principal difference was the absence of the electric street lighting which had become commonplace in my time. In the 1830s domestic electric lighting was nonexistent and houses were chiefly illuminated by candles and occasionally by gas. The streets were free of motor cars and people walked, rode a horse or were conveyed in a horse-drawn carriage to their destinations.

I knew that Paganini's unprecedented skill on the violin had gained him the reputation of being satanically inspired, some going so far as to claim that he had sold his soul to the devil in exchange for his transcendent technique on the fiddle. Paganini himself encouraged this suggestion, which he thought piqued the public into buying tickets for his concerts simply out of curiosity.

All this made me eager to see the demonic fiddler in action. On the night of the concert, I arrived at the theatre, which had recently been fitted out with gas illumination, to find it filled to overflowing. I took my seat and switched on my pocket recorder. The curtain rose to reveal a small orchestra (called a "band" on the playbill I had picked up on entering) which was quickly prodded by its conductor into a somewhat ragged rendition of a Weber overture. When this was over, the curtain fell, but soon rose a second time to reveal the star of the show, Maestro Paganini himself. I had expected the great man to be greeted by uniform cheering on the part of the audience, and was taken aback at the mixture of boos and hisses that accompanied his appearance. Evidently many in this crowd disapproved of his reputed satanism.

In appearance Paganini was tall, gaunt and thin almost to the point of emaciation. His cadaverousness was accentuated by the long black coat and trousers it was his habit to wear. His greying hair, long but sparse, fell over his shoulders. Taking up a stance, right leg slightly bent, he picked up violin and bow in his long, fleshless fingers, perfectly adapted for the violin, and immediately launched into a capriccio. His attack was ferocious, like a swordsman's. In *presto* passages the eye could not follow the speed of his fingers on the strings. In *cantabile* passages he coaxed a rich, dulcet tone from his instrument any coloratura soprano would envy. In arpeggiated passages his bow became an arched blur as it passed over the strings at lightning speed. It was obvious that much of his performance was improvised. I was staggered by the daring, the sheer outrageousness of his playing, made possible by his stupendous technical command of his instrument. Along with the rest of the audience, I was becoming intoxicated by his extraordinary virtuosity. The more extravagant, the more fantastic his playing became, the more his gyrations on the stage gave the impression of a man possessed. An odd, bitter smile flickered occasionally over his pale, sunken features as if to confirm his awareness of his possession. I indulged briefly in the fantasy that he was possessed, not by the devil, but by the violin itself. It was as if the fiddle were playing Paganini rather than the reverse.

At the end of the first part of the concert, the whole audience leapt to its feet and burst into wild applause. Paganini's playing had transformed the audience's initial hostility into unbounded enthusiasm.

When the concert resumed, Paganini performed several works of his own composition entirely on the violin's 4th (G) string. To enhance this display of pyrotechnics, Paganini had detached the three higher strings from the violin's tailpiece, allowing them to dangle conspicuously from its scroll. He ended the concert with a series of scorching variations on *God Save the King* . Wild applause again ensued.

Although Paganini was the star of the show, there were a number of other performers on the bill, including an eight-year-old girl pianist who showed a remarkable command of her instrument. In all, the concert had lasted more than three hours.

As I left the theatre after Paganini's spellbinding performance, I reflected on the remarkable events of his last few weeks. Even though I was becoming a seasoned chrononaut, I found it difficult to accept as real the experience of seeing and hearing in the flesh two of the greatest instrumentalists in history, whose birthdates were separated by more than a century. But I put these reflections aside and turned to the comparison of Paganini's and Heifetz's merits as violinists. First and foremost, it was obvious that both possessed stupendous techniques far beyond any other violinist I had heard. Both also had the ability to make the impossible seem easy. On that score I ranked them equal. Heifetz possessed a more polished, elegant sound and Paganini a fiercer attack, but the real difference between these two giants, I realized, did not lie in technique as such. The difference arose in their *raisons d'être.* Paganini had achieved fame at a time when virtuosity and instrumental showmanship were the *sine qua non* of musical performance, and thus a major reason for him to develop his remarkable technique was to put it on public display. Spontaneity and excitement, not perfection, were his primary goals. But this was not the case with Heifetz. While he was by no means indifferent to the public impact of his playing, his true ambition was to come as close to perfection as humanly possible, and to make it all seem effortless. In fact, I concluded, it was precisely the apparent effortlessness of Heifetz's playing, coupled with the sangfroid he invariably brought to his performances, that he wished to put on display. Yet, unlike the ascending lark, Heifetz's concept of perfection was not to be "seraphically free of taint of personality". On the contrary, the kind of perfection Heifetz sought *included* the brilliance and excitement that so moves audiences, only in a form of ultimate refinement. This is why he came to be called "God's Fiddler". But he didn't encourage this idolization. Paganini, on the other hand, was pleased to be called "The Devil's Fiddler", indeed playing up to the character in his performances. Paganini and Heifetz: two different personalities; two different *raisons d'être;* two glorious violinists with unsurpassable techniques.

Once I had returned to 1910 London, I started to plan my "interview" with Heifetz. Although Heifetz became very reclusive in his last decades, in earlier years he was quite communicative and willing to grant interviews. I thought a good time to attempt to meet with him would be during the nearly two year "sabbatical" he took from the concert stage starting in April 1947. I time-travelled to Los Angeles, arriving in November 1948, and I again took a room at the Ambassador Hotel. It turned out to be surprisingly easy to contact Heifetz - at that time he was listed in the telephone book. I dialled the number and after a few rings a female voice answered. "Mr. Heifetz's residence. Who's calling, please?" "I'm John Bellane, a senior editor with *The Strad* magazine of London. I'd like to speak with Mr. Heifetz." "Very well, I'll see if he'll speak with you." After a short pause, a voice, bearing a pronounced Russian accent, came on the line. "Heifetz speaking. My housekeeper tells me that you are with *The Strad.* Is that correct?". "Yes, that's right, Mr. Heifetz, John Bellane of *The Strad.* I'd very much like to conduct an interview with you, on informal lines, more of a conversation, in fact. I've no doubt our readers would be thrilled to read it." 'Well, I have to tell you I'm not very good with words and it might be best not to disillusion your readers. But all right, since I have a high regard for *The Strad* I'll do it. Could you come to my residence on Thursday at precisely 2 p.m.?" "Yes, I'll be there. Thank you very much, Mr. Heifetz."

I was aware of Heifetz's obsession with punctuality, so it weas at precisely 2 p.m. on Thursday that I knocked at the door of his Beverly Hills residence. It was opened by a woman, presumably the housekeeper, who ushered me into a sumptuously furnished room whose walls were covered with bookshelves and musical memorabilia. After a few seconds Heifetz, looking trim and relaxed, entered the room. We shook hands and I proffered one of the cards identifying myself as associated with *The Strad.* " Thank you very much for agreeing to see me, Mr. Heifetz. It's an honour." After giving my card a quick glance Heifetz said, "Happy to meet you, Mr. Bellane. I'd like you to accompany me to the studio I've recently had built. We can talk there." We left the house and walked the short distance along a covered breezeway connecting the house to Heifetz's studio. The building itself was an elegant redwood pavilion, hexagonal in shape. We entered through a small lobby which led to the studio proper, a spacious room dominated by a brick-and-stone fireplace. It had wide windows which overlooked the Beverly Hills canyons. The timbered slopes, dotted with cypresses and the red tile roofs of Mediterranean-style villas, could have been a Tuscan landscape. Heifetz had chosen an ideal location for his retreat.

In the room was a blue-green daybed, a desk bearing a bust of Beethoven, a black leather desk chair and an armchair. The built-in file cabinets occupying the room's back wall were festooned with musical memorabilia and cartoons, one of which caught my eye. It showed an irate customer complaining to the owner of a car repair shop: "$120.34 for a tune-up? Who tuned it? Jascha Heifetz?"

Heitetz waved me to an armchair. After a moment I remarked "What a charming room, and what a delightful view. One cam almost taste the tranquility. I imagine that you spend a lot of time here." "Yes, as much as I can", he replied. "I find it a wonderful retreat and an excellent place to practice. The building was only completed a few months ago. It was designed to my specifications by Lloyd Wright, Frank Lloyd Wright's son." Heifetz paused for a moment and said "May I offer you a drink?" "Yes, whisky and soda, please." "Of course. From your accent you must be an Englishman, so I take it you won't want ice. But please correct me if I am wrong." "No, no, you're right on both counts." Heifetz went to a built-in cabinet in a corner of the room and opened its doors to reveal a well-stocked bar. He mixed two whiskies and soda and handed one to me Seating himself in the chair behind his desk, he said, with a touch of jocularity, "Well, what shall we talk about? Cabbages and kings, maybe? "

"Many things, I trust," I rejoined. "But I would like to start by asking you, as the violinist all others revere, what do you regard as being the most important attributes of a musical artist?"

Looking slightly taken aback by the question, Heifetz responded, "Well, I thought you'd start by asking me whether that Elman -Godowsky story was true. So give me a second to think about your question." He paused for a moment, and then answered: "I should say that the most important attributes for a serious musician are: self-respect, musically speaking, integrity, and enthusiasm."

"Well said, Mr. Heifetz. But you weren't far off in your expectation. I intended the authenticity of the Elman-Godowsky anecdote to be my second question."

"Aha, I thought so!" Heifetz rejoined with some glee. "All right, I'll tell you. That story is perfectly true. On the night of my New York debut, I remember, October 27, 1917, Godowsky came to me during the intermission and told me the story. It's been dogging my footsteps ever since. What amuses me is the sequel to the story, and a pianist is the target this time:

"At a concert in London one summer night, a famous pianist was listening to Jozef Hoffmann play the piano. He tried to take his handkerchief out of his pocket and mop his forehead - but he noticed that several of us, violinists, were looking at him, with the obvious remark of Godowsky on our lips, and he placed his handkerchief sheepishly back in his pocket and sweated it out!"

"Ah, the violinists' revenge. Perfect symmetry," I said with a chuckle. "Another famous story about you is that after attending one of your concerts Bernard Shaw wrote to you with the advice to play one wrong note every night so as not to provoke a jealous God."

Assuming a deadpan expression, Heifetz said, "Yes, he did. I still have the note. And I took his advice very seriously. Just to be on the safe side, I started playing *two* wrong notes every night."

"Very funny," I said, laughing. "But seriously, do you think that there's a rivalry between violinists and pianists?"

Heifetz lit a cigarette, and pondered the question. "Well, you know, when I was a boy I was more attracted to the piano than the violin. But my father was a violinist and wanted me to follow in his own footsteps, so that's what I became. But I've always loved the piano and admired good pianists. I play the piano myself, you know, but I don't think Horowitz is worried. You ask me whether there's a rivalry between violinists and pianists. I think there is, but only at the level of competing for audiences. As instruments the violin and the piano are as different as, what is the phrase, chalk and cheese. Even though the sounds that come out of both obey the laws of harmony."

"Yes, that's true," I rejoined. "One important way in which the piano differs from the violin, along with all bowed and blown instruments, as well as the singing voice, is that notes on a piano are produced individually and distinctly, discretely if you will, while in the case of the violin and the others the notes are drawn out, continuous. This difference between the violin and the piano is the main reason why the two instruments don't 'blend' in a sonata in the way that the instruments in a string quartet, or the voices in a choir, can combine to produce a homogeneous sound."

Heifetz gave me a quizzical look. "Are you a professor or a magazine interviewer, or what? But yes, what you've said is right, although I wouldn't have put it so academically. I've always thought that the piano is the ultimate instrument for using the fingers of both hands. With the violin only the fingers of the left hand are fully used, the fingers of the right hand are only used to hold the bow. It's really the right arm, not the fingers of the right hand, that actually controls the bow."

I nodded in agreement. "Yes, exactly. And at a somewhat lower creative level, the skilled typist also maximizes the use of the fingers. But do you mind if we move on to another topic? I've always wanted to know about your relationship with Schoenberg, and whether you really declared his violin concerto to be 'unplayable'".

Heifetz grimaced. "I'm sorry to say that my relationship with Schoenberg has not been the best. Oscar Levant told me, in 1937, I think , that Schoenberg had composed a violin concerto and wanted me to play it. He was hoping that we might get together so that I could take a look at the score. I arranged to meet Schoenberg, he presented me the manuscript score of the concerto and I started to play through the violin part. It was very thorny and quite frankly I didn't understand, nor like, the work. At one point I arrived at an especially tangled passage and protested, in a humorous way, 'Sir, hasn't it occurred to you that one needs six fingers in order to play this?'. To this he responded, in a way I thought to be in the same spirit, "Well, I can sit and wait until you grow one."At that point we decided to give it up, and we parted on what I thought were friendly terms. But later I heard that he was offended, and then pleased when the rumor went around that 'Heifetz declares Schoenberg's concerto too difficult for him to play.' As a matter of fact I was pleased too, because it got me off the hook of actually having to play the thing. But I still regret the incident. Schoenberg had a very hard time after having to get out of Germany when Hitler took over and I wanted to help him. I just didn't want to play his concerto, it felt alien to me."

"Yes, I suspected as much," I replied. "It didn't seem possible to me that Schoenberg, or indeed anybody else, could compose something humanly possible to play but which could defeat you. Especially since Louis Krasner later performed it successfully."

"Yes, he did," Heifetz said with a shrug. "But only if by 'successfully' you mean 'struggling through it without dislocating a finger.' But I'm being unfair to Krasner. He's an excellent violinist, only we don't share the same tastes."

"Now I hope" I said , "you won't mind my broaching a subject that I know you've been asked about on numerous occasions, and that is the lack of emotional expression, in comparison with most other players, you show when you perform. Some have even called you 'the great stone face.' This sangfroid is very much part of your performing style, and I'm guessing it must be at lest partly deliberate. Is this correct?"

"I was bracing myself for that one," Heifetz said with a rueful grin. "I'm often asked that question, and I have come to reply by saying that I believe that when an instrumentalist performs, he or she is acting as an extension of the instrument itself. The sounds emanating from the instrument convey emotion directly to the listener, who may shed tears, go into ecstasies, or whatnot. But the instrument does not itself show emotion: it acts as a vehicle for conveying emotion. I think that the instrumentalist, as an extension of the instrument, is in the same position, that is, is acting fundamentally as a means of conveying emotion through sound alone. Showing a lot of emotion through exaggerated facial expressions, body swaying, waving the instrument about, and the like, seems to me superfluous and not part of the purpose of playing. When I play, my inner feelings are often turbulent, but since my personal feelings are not what I'm trying to convey I keep them to myself. Of course, the composer's feelings are another matter altogether! Anyway, that's my philosophy. You can take it or leave it. "

"You've made your position admirably clear," I responded. "In your composure I see not a lack of emotion, but a remarkable degree of self-discipline, the kind of self-discipline required by an expert billiards player or a tightrope walker."

"Yes, that's a good analogy," Heifetz agreed. "And I do believe self-discipline is an essential attribute in an artist, which I should have included in my answer to your very first question."

"If I may, "I said, "I'd like to ask you about your 20th century concerto repertoire. You've performed and recorded a number of 20th century violin concertos: Elgar, Sibelius, Prokofiev no 2, Korngold, for example. You've commissioned concertos by Walton, Castelnuovo-Tedesco and Gruenberg. But there are some notable omissions, works which I personally would adore hearing you play, Prokofiev no. 1, Stravinsky, Barber, Berg, Bartok, Hindemith 1939 for example. Could you explain to me why you never recorded (or, as far as I know, ever played) these works?"

"Well, that's a good question," Heifetz rejoined with a thoughtful expression. "Let's see, I liked, and still like, Prokofiev's First, and I probably would have taken it up but Szigeti beat me to it. It became an important part of his repertoire and I didn't want to compete with him. Did you know, by the way that in 1923 Milstein performed the concerto in the Soviet Union with Horowitz playing the orchestral part on the piano? They were both 19 years old at the time."

"No, that's new to me."

"I actually like the Stravinsky concerto. "Heifetz continued, "But after the work came out in 1931, Samuel Dushkin, the work's dedicatee, had a five-year performance exclusivity on it, which frankly annoyed me, and as a result it never entered my repertoire. As for the Barber concerto, I heard Albert Spalding give the first performance in 1941, and I thought it a beautiful piece, the first two movements anyway, the last movement, that crazy *perpetuum mobile*, is interesting in itself, but somehow doesn't belong with the other two. I might have taken it up, but the war interrupted everything and when that ended lots of violinists jumped on it and I decided to leave it alone. In the case of the Berg concerto, although it's composed in Schoenberg's twelve-note style, it's actually quite lyrical, even beautiful in parts, but I thought I'd leave it to the atonalists. As for the Bartok concerto, Tossy Spiakovsky took it up after the war, and he did such a beautiful job with it that I thought I wouldn't get in his way. Finally, I was never attracted to Hindemith's music, it sounded mechanical to me."

"You've been very patient, Mr. Heifetz," I said, "But please allow me to put one final question to you. There is a famous characterization of Russians attributed to you: one Russian is an anarchist, two Russians a game of chess, three Russians a revolution, four Russians the Budapest String Quartet. Was this your invention? "

"I can only take credit for 25% of it, "Heifetz said with a smile. "It's like this. One evening, it must have been sometime in 1938, I was playing bridge with my friends Oscar Levant, and Isidore and Joseph Achron. At some point we paused to take refreshment and Oscar suddenly remarked, 'You know, apart from me, all of us here are Russian-born, and my parents were Russians. Let's play a little game. How would each of us define a Russian? Better still, let's think of how we'd define one, two, three and four Russians. I'll start with one. One Russian is an anarchist. Isidore, Joseph and Jascha, could you do two, three and four?' Isidore thought for a while and said 'Two Russians are a game of chess.' Then Joseph came up with 'Three Russians are a revolution.' That left me really on the spot. How was I going to follow these clever answers? Then suddenly a light bulb went on in my head. 'Four Russians are the Budapest String Quartet,' I said. It got a good laugh, I can tell you."

"Mr. Heifetz, that's a really delightful story, and a wonderful way to end the interview. Thank you very much. I've greatly enjoyed our conversation."

We both stood up and as they shook hands Heifetz said, "Mr. Bellane, it's been a pleasure. But I've noticed that you haven't taken any notes. You must have an excellent memory."

"Mr. Heifetz, I've got total recall, I replied.

George Sanders

Early in my career as a chrononaut I met with George Sanders, the Russian-beorn British actor. His portrayals of urbane, cynical, witty, world-weary characters in movies of the 1940s and 50s had made a great impression on me, in particular his role as Lord Henry Wotton in the 1945 movie of Oscar Wide's *The Picture of Dorian Gray* and, above all, as the self-serving, acerbic theatre critic Addison DeWitt in 1950's *All About Eve,* a role for which he won the Oscar for best supporting actor. I was curious to learn whether George Sanders the man was as witty and urbane as the characters he played on the screen.

My researches into the future had revealed that in 1972 Sanders committed suicide, leaving the note "Dear World, I am leaving because I am bored. I feel I have lived long enough. I am leaving you with your worries in this sweet cesspool. Good luck." I have long respected suicides for their courage in terminating their lives, quite independently of other aspects of their characters, however base or noble. While I understand – although scarcely agree with - the Catholic view that suicide is a sin, the secular view that it is an act of cowardice has always baffled me. I concur with Dostoevsky that the urge to continue living is extremely strong, and as a consequence I believe that that it requires great courage to go against the grain and commit suicide. I also know that I do not possess the courage to do away with myself. Long ago I observed that *an act of suicide has the effect of trivializing the lives of those who live on*. It occurred to me that, in choosing to meet George Sanders, a man whose message to the world on his departure from it was freighted with such cynicism, I was unconsciously protecting myself from the pathos I would surely have felt at knowing in advance the date of death of someone less worldly.

I selected November 1954 as the target date for meeting Sanders, who was living in Los Angeles following his divorce from his second wife, the Hungarian actress Zsa Zsa Gabor. My arrival in Los Angeles went smoothly and was without incident. I booked a room at the fashionable Ambassador Hotel, famous for its popularity with celebrities, later to become infamous, as I knew, as the site of the 1968 assassination of Robert Kennedy. As I entered the hotel, I heard a bellboy shout "Mr. Douglas, your taxi has arrived!", and saw a man, instantly identifiable as Kirk Douglas, rise from a chair in the lobby and march smartly out the establishment's ornate double doors. "There's another character I'd like to 'interview' ," I thought. When I was shown to my room I was pleasantly surprised to find it to be tastefully decorated, far from the gaudy style I'd expected. I had room service send up a gin-and-tonic and got down to the business of contacting George Sanders. The simplest way of doing that was to engage the services of a private detective to find out Sanders' current address and telephone number. The phone book revealed several columns in its classified section listing private detectives. I scanned the names starting with "M", indulging in the whimsy of finding "Marlowe, Philip". I was startled to find a Philip Marlowe actually listed, along with an address and telephone number. I picked up the phone and dialled the number. After a few rings a sultry female voice answered "Marlowe detective agency, can I help you?". "Yes, I expect you can. Might I speak with Mr Marlowe?" "I'll try to connect you." After a short pause, another voice, gruff but businesslike, came on the line. "Marlowe speaking. What can I do for you?" "Well, Mr. Marlowe, I'd like to engage your services in pursuing a matter of no great complexity but perhaps of some delicacy. I'd rather discuss it with you in person, if possible." "That's fine with me. Why don't you come to my office tomorrow morning? The address is in the book. Can you drop by around 11? By the way, what's your name?" "That would suit me perfectly, Mr Marlowe. And my name is John Bellane."

The next morning I presented himself at Marlowe's office, located on the sixth floor of the Arbogast Building, a weathered edifice in downtown Los Angeles. The pebbled glass door: to the office bore the inscription

*Philip Marlowe Investigation*s
Trouble is My Business!

I opened the door to an outer office in which an attractive secretary sat at a desk agitating the keys of a bulky typewriter. She reminded me of Lucille Ball in the movie *The Dark Corner.* As I entered the office she looked up, dropped her hands from the keys, and with an enquiring glance, said "What can I do for you, sir?" "I'm Mr. Bellane and I have an 11 o'clock appointment with Mr. Marlowe." "Oh yes, of course. He's expecting you. Go right in." She gestured at a door at the rear of the room. The door suddenly opened and a short, rotund character with thinning red hair thrust himself through. I was irresistibly reminded of Carroll O'Connor in *Point Blank.* He looked me over and said:

"Mr Bellane, right? I'm Marlowe. I've been expecting you. Come in, come in! Sit yourself down and tell me your troubles. You probably noticed I regard trouble as being my business."

"Well, Mr Marlowe," I replied, "in all honesty I don't think that what I seek to engage your services for deserve to be called 'trouble'. Nevertheless I believe that you'll be able, and I hope willing, to help me."

“OK, let’s hear it.”

“I want you to put me in touch with George Sanders.”

“You mean that smooth actor with the upper class diction?’

“Yes, that’s the chap.”

‘’So, what do you want me to do, introduce you to him personally?”

“Nothing so exacting. All I desire is that you determine his present whereabouts, and furnish me with his address and telephone number.”

“Is *that* all you want me to do? Hell, that’s a milk run. It’ll cost you 50 bucks. Now I don’t ordinarily ask a client about his reasons for employing me, but in your case, Mr Bellane, I’m going to make an exception. So tell me why do you want to contact George Sanders?”

“I’ll be happy to tell you, provided that you agree to answer some questions I’d like to put to you.”

“ OK, it’s a deal.”

‘’I’m a writer employed by the movie magazine *Photoplay* seeking to interview Mr Sanders.

“ Well, that makes sense. All right, shoot your questions to me.”

“First of all, is Philip Marlowe your real name? I have to say I doubt it. And did you take the slogan ‘Trouble is my business’ from Raymond Chandler?”

“OK, you got me. I could tell you’re a man with a head on your shoulders. My birth name was Bernard Schwartz, not that I’m ashamed of it, mind, but when I went into the private eye business I figured I’d get a better class of client if I put my favorite detective writer Raymond Chandler’s famous private eye Philip Marlowe’s name, along with ‘Trouble is my Business’, my favorite title, on my shingle. I’d never stoop so low as to use the name of that bum Mike Hammer!”

Handing the detective a fifty dollar bill, I said, "I'm delighted to hear that. It confirms that I'm entrusting my business to the right agency. I'm staying at the Ambassador hotel. Please inform me when you have the information I've asked for. Mr Marlowe, you are a gentleman and Mr. Schwartz, you are a scholar. It's been a pleasure meeting both of you." I got up, shook hands with him and made my departure.

The next day a message came through with George Sanders' current address and phone number. I picked up the phone in my hotel room and muttering "jacta alea est" under my breath, dialled Sanders' number. The phone continued ringing for what seemed an inordinate length of time. I was about to give up when a languid voice, unmistakably that of George Sanders, came on the line. "Yes, what is it?" "Am I speaking to Mr George Sanders?" "Yes, regrettably, you are. What is it you want?" "I apologize for disturbing you, Mr. Sanders. My name is John Bellane and I'm a senior editor at *Photoplay* magazine. I'd very much like to conduct an interview with you, which I hope would take the form of enlightened conversation conducted in civilized surroundings." "Well, I don't normally agree to interviews as the published ones are so vulgar. But your proposal, Mr. Bellane is it? , of a civilized exchange I find appealing, and certainly unusual in this distinctly uncivilized environment. All right, I'll risk it." "Excellent. When would you like our rencontre to take place?" "Well, let me see. I'm free the day after tomorrow in the morning."

We agreed to meet at 11 a.m. two days later in a quiet bar of Sanders' choosing. To make myself easily identifiable, I had told him that I would sport a red carnation in the buttonhole of my jacket. I arrived at the agreed time but there was no sign of him. I sat in a booth at the back of the bar, summoned the waiter and ordered a gimlet. The waiter had not heard of the drink and I explained to him that it was simply a combination of equal parts of a good English gin and Rose's lime juice. And please, I added, no ice! The waiter wasn't sure whether the bar had Rose's lime juice, but he thought there might be an unopened bottle knocking around somewhere. He went off and after a while returned bearing a bottle of Gilbey's gin, a champagne glass and, yes, a dusty unopened bottle of Rose's lime juice. "I figured you'd like to mix this yourself," he remarked with a self-satisfied air , "seein' as you're so partic'lar." I thanked him and proceeded to mix my drink.

Half an hour went by before Sanders finally appeared. He spotted me with my buttonhole and came directly over to my booth. He was a tall man, his hair smoothly slicked back.

He wore a sporty Harris tweed jacket and an ivory-coloured cravat. He looked every inch the urbane gentleman of the old school. As he approached, I surreptitiously switched on my pocket recorder and stood up. We looked at one another for a moment in silent assessment. Then he said, in the polished tones with which I was familiar from his screen performances,

"Mr. Bellane, I presume? I'm so sorry to be late. If memory serves, it was Louis XVIII who observed that punctuality is the politeness of kings, in which case I have also to offer you an apology for not being a member of that privileged club."

"'No apologies necessary, " I returned. "As a fellow non-member of that club, let me say that it's a great pleasure to meet you."

"You are most gracious," Sanders replied with an engaging smile.

Sanders sat down opposite me and, noticing the bottles arranged on the table, remarked,

"Aha, I see that you're indulging in a gimlet. It happens to be my favourite drink."

"Good, I'll ask the waiter to fetch you a glass so that you can mix your own."

The glass arrived and he mixed his gimlet and began to sip it with evident satisfaction. Then he pulled out a cigarette case, along with a holder, from an inside coat pocket and offered me a cigarette, which I declined. He inserted a cigarette into the holder and lit up with an elegant gold lighter which seemed, miraculously, to have materialized from nowhere. He then looked across the table at me and asked, companionably,

"Have you read Ray Chandler's latest effort, *The Long Goodbye,* by any chance?"

"I have, as it happens."

"Well, in that case you'll recall that in the novel the gimlet is Terry Lennox's tipple of choice, and he introduces Marlowe to it."

"Yes, I remember the episode. I take it, then, that you like Chandler's work?"

“Yes, very much,“ Sanders affirmed. “I’ve met him a few times at parties. I recall that he drank rather too much, and that made him unpleasantly garrulous. But who am I to criticize someone whose work I admire without reservation?” He waved his cigarette holder in what I took to be a self-deprecatory gesture.

“With your permission “ I said, “I’ll don my interviewer’s hat and ask you to comment on some of the roles you have played on the screen.”

“Very well,“ Sanders replied with a resigned air, “but please remember it was you who laid down the condition that our exchange must remain within the bounds of civility.”

“Of course,“ I responded. “You will hardly be surprised if I begin with what is surely your most memorable role, for which you received an Oscar, that of Addison DeWitt in *All About Eve.* The nature of the character is established right from the start with your sardonic off-screen declaration, which I hope you’ll permit me to quote, since I enjoy quoting it:

To those of you who do not read, attend the theatre, listen to unsponsored radio programs, or know anything of the world in which you live, it is perhaps necessary to introduce myself. My name is Addison DeWitt. My native habitat is the theatre. In it, I toil not, neither do I spin. I am a critic and commentator. I am essential to the theatre.

Now I have no doubt that you’ve been told on numerous occasions that the role fitted you like a glove, or some similar expression. Do you think this is true, and do you believe that you personally share some of the attributes DeWitt displays on the screen?”

Sanders mixed himself another gimlet, and, downing it quickly, treated me to that arch expression so familiar from his movies. “My dear fellow,“ he said in his best upper-class drawl, “Let me first congratulate you on the retentiveness of your memory. You might have made a good actor. But to answer your questions. I am myself an actor after all, and if I may say so without sounding conceited, a reasonably competent actor, although I don’t have an especially high regard for the thespian profession itself. So even in those cases, and, as with many actors, my career is replete with them, where the nature of the character I attempt to portray is very different from what I consider to be my own inner nature, I still try, as a matter of professional pride, to turn in the best performance I can. But in the case of Addison DeWitt, whose dialogue was crafted (along with those of all the other characters) with such brilliance by Joe Mankiewicz, I have to admit that I admired the character's intelligence, articulacy, mordant wit, and elegant style sufficiently to feel flattered when I was told that the role 'fitted me like a glove'. "

He paused to insert another cigarette in his holder. "Of course," he resumed, "those making that observation may have meant not just those aspects of DeWitt's character I admired, and believed to some extent I shared, or at least aspired to share, but also the other less savoury, indeed downright despicable aspects of his character that I emphatically hope I do not share, for example his utter selfishness, his withering contempt for other people, the overweening conviction of his own superiority. Nevertheless I have to admit to succumbing in some degree to a feeling of pride in my own portrayal of those aspects of DeWitt's character that I thoroughly detest. "

"Very nicely put, sir," I rejoined. "But don't you think it understandable that some of your devotees choose to believe that you share certain of DeWitt's less admirable qualities? After all, you have publicly referred to yourself as a "cad" and one of your most famous lines, from the movie *Lured,* is 'I'm an unmitigated cad.'"

Sanders chuckled. "That's true. But both in my public utterances, and in Sirk's movie, the observation was made with ironic intent. I must admit, though, that irony is a rhetorical device that both DeWitt and I enjoy using. In fact, if I ever write an autobiography, I shall have no hesitation in entitling it *Memoirs of a Professional Cad.*[1] My public will be delighted."

"If you'll allow me," I said after a brief pause, "The other role of yours I'd like to discuss is that of Lord Henry Wotton in *The Picture of Dorian Gray.* A few years after the original novel's publication, Wilde wrote to a correspondent,

Basil Hallward is what I think I am: Lord Henry what the world thinks me: Dorian what I would like to be – in other ages, perhaps.

I presume that you must have read Wilde's novel. But were you familiar with this quotation when the film was made?"

"Naturally, like every literate anglophone, I'd read the novel, " Sanders replied with a shrug. "But no, until you mentioned it, I'd never come across Wilde's quotation. Of

[1] *Sanders did follow through with this, publishing an autobiography with that title in 1960.

course, it's clear from all those witty epigrams issuing so smoothly from Lord Henry's lips that he was intended by Wilde to serve as a portrait of Wilde's public persona, the brilliant, showy, 'I have nothing to declare but my genius' side of his nature. As for the movie, I have no high opinion of it, and while I enjoyed delivering all those *bon mots*, I thought my portrayal of Lord Henry rather superficial, in no way comparable with the effort I put into bringing flesh and blood (if that's what it was, it seemed like ice to me) to the role of Addison DeWitt. This is a pity, because the one character I've always had a yen to portray is Oscar Wilde himself. By the way, I've also longed to play Algernon Moncrieff in my favourite play, *The Importance of Being Earnest,* but, alas, I have never been offered the part. I did play Lord Darlington, however, in a mediocre film version of *Lady Windemere's Fan.* From my youth I've admired Wilde's epigrammatic brilliance, as well as the depth of his thinking, which isn't nearly as well known. This may be attributable to the scandalous reputation he acquired through his sexual proclivities. I don't share them, , but I certainly don't condemn them, and I think it monstrous that because of them he was he was treated so shamefully by the society of his day."

I refilled Sanders' glass and asked him, "What is your opinion of Wilde's aestheticism, his declaration of *ars gratia artis,'*art for art's sake'?"

At this point Sanders burst out laughing. "Let me tell you what I consider to be the ultimate absurdity of the modern world. It is the fact that on every film produced by Metro-Goldwyn- Mayer you will find emblazoned the motto *are gratia artis.* Fortunately, Oscar Wilde's refined sense of humour has undoubtedly been inherited by his shade so that, far from spinning in its grave, it's dying of laughter, if that is possible for shades. But to address your question. The received opinion of Wilde was that he was a thorough cynic. I don't believe this for a moment. I believe his cynicism to have been something of a pose, an attitude cultivated as a form of self-protection during his years at Oxford, but which he employed so successfully that it became ingrained. I think that, as a serious philosopher, and he was that, you know, although he'd be the last to admit it, even to himself, perhaps…. as a serious philosopher he truly accepted the doctrine of aestheticism. On the other hand, I think that Dorian's macabre fate shows that Wilde drew the line at pure hedonism: *delectatio gratia delectationis* if I recall my school Latin. Now there's an appropriate motto for MGM! Or better still, *pecunia gratis pecuniae.* I apologize for succumbing to the temptation to show off. In that respect I certainly do resemble Wilde, or Lord Darlington at least, who, as you will doubtless remember, declared that he could resist everything except temptation. But to repeat, I think Wilde was a genuine aestheticist, if that is the word. As for myself, while I love beauty of all kinds, I have to admit that my innate cynicism inclines me to hedonism, but only of the etiolated sort embodied by Lord Henry. Does that answer your question?"

At this point I rose and said, "Mr Sanders, I think you've answered all my questions admirably, with the penetration and wit I fully expected. I thank you very much for your graciousness in allowing our discussion to take place. It has been a great pleasure for me, as well as most informative."

Sanders also rose, and raising his glass, declared "I confess that I did not expect our tête-à-tête to be so stimulating. But it has been, and I thank you, Mr. Bellane, for the pleasure it has given me. One thing puzzles me though. How is it that you've not found it necessary to take notes?"

"I have total recall," I said with a grin.

As I left the bar I suddenly realized that Sanders had not bothered to ask me to show my credentials.

Alexander Grothendieck

Of all 20th century mathematicians, I was most intrigued by Alexander Grothendieck. I had learned that Grothendieck was regarded by many later 20th century mathematicians as their greatest contemporary, perhaps the greatest of the century. Born in Berlin in 1928, Grothendieck spent his adolescence in France. In the early 1950s he erupted into French, and world mathematics "as if from the void", revolutionizing algebraic geometry and in the process creating a vast new kind of mathematics unifying the continuous and discrete. In 1966 he was awarded the Fields Medal, the most prestigious prize in mathematics. In the 1960s his social conscience began to play an active role in his life, leading to his participation in pacifist, antinuclear, and environmentalist causes. He refused to attend the Moscow Fields Medal award ceremony as a protest against the political imprisonment of writers in the Soviet Union. By 1969 he had embarked on a campaign of protest against the military-industrial excesses of contemporary society, which he believed, if left unchecked, would lead to the destruction of human society and the whole of the natural world. His critique was sharply directed at science, which he singled out as playing a central role in advancing the military -industrial agenda. In his view, science had become a new kind of religion which he termed "scientism". He had come to see mathematics as part of the hieratic language of scientism. This led him to retreat from professional mathematical activity. In 1970 he resigned his professorship at the Institut des Hautes Etudes Scientifiques (IHÉS) following a dispute over military funding. In the first years of the 1970s he made a concerted effort to get others, in particular his mathematical colleagues, to join him in his campaign against the establishment. Disappointed with the results of his efforts, by 1973 he had retreated to Villecun, a small village near Montpellier in the south of France. I learned that Grothendieck had for several years occupied a house in the village where he led a simple, even spartan life, subsisting on produce from the local communes or individual farmers. He did not sleep in a bed, but on a door detached from its hinges. He had also jettisoned his radio and cut off the electrical power, using a petroleum lamp for illumination. He welcomed everyone into his house, villagers, personal friends, colleagues and passing strangers alike. For a time he taught at the University of Montpellier, where he had studied as an undergraduate. Later he moved to several other isolated places in the south of France, maintaining a correspondence with ex-colleagues and friends, and writing his voluminous "Meditations", amounting to many thousands of pages, which are devoted to mathematical, biographical, religious, esoteric, and philosophical themes. These include *Recoltes et Semailles,* an extended reflection on himself and his life as a mathematician, and *La Clef des Songe*s, a meditation on the nature of dreams. In this latter work Grothendieck describes his revelation that dreams are messages sent by a power he calls the Dreamer and whom he identifies as the Deity. These messages, he declares, are sent in order to enable human beings to understand themselves and to find their way to a true life.

In 1991 Grothendieck vanished abruptly. Only much later it emerged that he had withdrawn to a small village in the Pyrenees where he lived in virtually total solitude until his death in 2014 at the age of 86.

Given all this, it seemed to me that the right time and place to approach Grothendieck was 1975 in Villecun. My first move was to time-travel to New York City on 24 June 1993 and 9 August 2020 to purchase copies of those two days' issues of the *New York Times*. I had ascertained that the front page of each these issues contained an article of significance for my purposes. The first of these, headed *At Last, Shout of 'Eureka!' In Age-Old Math Mystery,* reported that Fermat's Last Theorem had finally been proved by Andrew Wiles, an English mathematician working at Princeton**.** The proof was based on Grothendieck's work. The second article, headed *A Hotter Future is Certain, Climate Panel Warns* announced that a number of devastating impacts of global heating due to increased levels of carbon dioxide in the atmosphere were now unavoidable.

On my return to London, I detached and folded the papers' front pages, put them in an envelope, and sealed it. Then I wrote on it "Pour Alexander Grothendieck, certains temoinages de l'avenir". I intended to leave this envelope with Grothendieck, even though I was fully aware that in doing so I would be violating my self-imposed "chrononaut's code". But I thought the risk worth taking and minimal in any case. I put the envelope in his inside pocket of his jacket, picked up the Liberator, and set it to take me to Villecun on June25, 1975. I arrived to find myself in a tiny village, no more than a clutch of modest houses, perched on a hillside a dense wood. I came across one of the locals and asked him where "le grande mathematicien" lived. He gestured towards the smallest and least inviting of the houses that I could see.

I walked the short distance to the house and knocked tentatively at the door, which was partly open. There was no reply, so I pushed the door open a bit further and peered round it. I saw a long, dimly lit room with a table at one end, beyond which I could discern a number of cardboard boxes each overflowing with papers. At the table there sat a bald, ascetic looking man dressed in simple khaki shorts and tunic. He was bent over the table working away at a manuscript, his pen moving smoothly and rapidly over the paper like a violinist's bow over the strings. In his absorption he had noticed neither my knock nor my presence at the door. Feeing a stab of guilt at puncturing the evident concentration the man brought to his writing, I ventured to say to him, in French, "I'm sorry to disturb you, but could you tell me if I am correct in my belief that you are Alexander Grothendieck, the illustrious mathematician?"

His pen suddenly stilled, and, looking up at me with a quizzical expression, he replied also in French, "Yes, I am Grothendieck, the mathematician, or, more exactly, the ex-mathematician. And yes, at one time I can say without immodesty that I was considered illustrious. So now I'm ex-illustrious. But I still try to be industrious, at least graphically, as you can see. And who might you be?"

"My name is John Bellane. I'm honoured to meet you, sir."

"And now that we've met", said Grothendieck, with renewed perplexity, "May I ask if your visit has a further purpose? Are you, perhaps, a mathematician seeking to discuss mathematics?"

"Well," I responded, " I was originally trained both as a mathematician and a musician. Mathematics and music still rank very highly among my preoccupations. But over the past several years I have become a kind of pilgrim, roaming the globe in search of insight. In particular I seek out remarkable persons so as to engage them in discussion and thereby learn their worldview. You are unquestionably the most original mathematician of our time, and a man of strong convictions and deep moral integrity. So I've sought you out in the hope that you will be willing to talk with me."

Grothendieck sat in silence for a while and then said, "I've had a number of visitors here with whom I've discussed all sorts of topics, from mathematics to the current deplorable state of the world. Your forthrightness and evident sincerity appeal to me and I'd enjoy talking with you. So please pull up a chair and sit down. Will you join me in some tea?"

Delighte by this cordial response, I accepted the offer with thanks and sat down at the table. Grothendieck had meanwhile withdrawn into the house's small kitchen where he could be heard preparing the tea. He soon reappeared, holding a wooden tray on which rested an earthenware teapot and two handleless Japanese cups. He sat down at the table, poured tea into the cups, passed one to me, and, raising his cup, said, "In the words of my Japanese friends, Kinko ni aru! Now, since colloquy is the avowed purpose of your visit here, I invite you to initiate it."

"You are most accommodating, sir," I responded. "I would like first to ask you a question which has doubtless been put to you on numerous occasions: what is it about mathematics that attracts, or attracted you so strongly?"

"Ah yes," said Grothendieck, "I've often been asked that question, but each time I find myself wording my answer differently, even though the essence of my reply is always the same. When I first began to learn mathematics as an adolescent I was attracted by the clarity of its concepts, the certainty of its propositions. I soon became convinced that this clarity could be refined into complete transparency, so that all hidden meaning of mathematical concepts is revealed and becomes fully exposed to the mind. I saw that the way to achieve this ideal is through the development of generality. The correct degree of generality in mathematics should, ideally, create a perspective within which difficult individual mathematical problems come to be seen as special cases of essentially simpler problems formulated within an enlarged, general setting. Here the purpose of generalization is simplification. This was the aim of Descartes and Fermat in creating algebraic geometry, of Galois in his approach to the theory of equations, it is the aim of the creators of abstract algebra and category theory, and it is, or was, the goal of the Bourbaki fraternity. I saw all of these as instances of a general procedure in which a large number of scattered objects are united by building a suitable framework within which they can be displayed so that their hidden unity becomes manifest."

Grothendieck paused for a moment to refill his cup. Then he went on: "This idea, or metaphor if you like, drove my mathematical work, which was mainly in functional analysis and algebraic geometry. But I came to adopt a different metaphor. I now liken the effect of applying the right sort of generality to an intractable mathematical problem to the effect of immersing a hard nut in water or another liquid to soften it. From time to time one may rub it so as to improve the liquid's penetration, but basically one simply allows time to pass. It may take months, but gradually the nutshell becomes more and more pliable. Eventually a point is reached when the shell has become so soft that merely squeezing it causes the nut to open like a ripe avocado. Recently another metaphor occurred to me. I envisaged the unknown thing I wished to know as an expanse of earth or hard marl, resistant to penetration, at some distance from the sea. Over geological time the sea advances silently and imperceptibly, yet it eventually engulfs the resistant substance."

“Those are powerful and arresting metaphors,” I said. “The first strikes me as being very much in the spirit of *homo faber*, that human beings must engage in an unceasing effort to attain a goal. The other two metaphors seem to betoken the opposite idea, namely that, after appropriate preparations have been made, one needs just to wait patiently for a desired result to be achieved. The physicist Eddington once described mathematicians as ‘writhing before the idol of proof.’ While you evidently share with your fellow-mathematicians the view that proof plays an indispensable role in mathematics, your use of these metaphors seems to show that, far from idolizing, or fetishizing the proof concept, let alone writhing before it, your ambition was always to transform its fearsome aspect into something benign, even routine.”

Grothendieck nodded. “Yes, lurking in the back of my mind was the idea of constructing a universal framework for mathematics equipped with sufficiently powerful conceptual tools to make it almost, if not quite obvious, how to prove individual propositions. In this Utopian world the proofs of mathematical propositions might be lengthy, but each step in a proof would be as clear as the next step in climbing a ladder. As for the metaphors, I use the Taoist terms: the first metaphor is *yang,* the second and third, *yin.* Fundamentally, it was the transformation in my outlook from *yang* to *yin* that led me to abandon the mainstream professional mathematical world and retreat to this quiet village. I should add, however, that I still teach mathematics, mainly at an elementary level, at my first university in Montpellier.”

“There were, I understand, certain key external events that played a role in the transformation of your outlook. Would you be willing to describe some of them?”

Grothendieck laughed and said, “You originally proposed a discussion, but this is beginning to resemble an interrogation! But all right. Throughout the 1950s and early 1960s, I was the perfect embodiment of *deformation professionnelle,* devoting myself body and soul to mathematics, and to my conviction of how mathematics should be done. During this time the pursuit of my mathematical vision was almost fanatically single-minded. Still, this obsession with mathematics had not effaced the memory of my childhood of my parents, both of whom were champions of social justice, and who suffered greatly during the German occupation of France during the Second World War. My father, who was Jewish, was sent to Auschwitz and was murdered there. My mother and I were incarcerated in internment camps in occupied France. These monstrous injustices I never forgot, and it is doubtless for this reason my sympathies have always lain with the persecuted and oppressed. If I could claim to have had political convictions, I would now characterize them as anarchist. But for a long time these feelings were latent, mere undercurrents to the blinding pursuit of my mathematical ambitions. True, along with most decent people, I opposed the French colonial war in Algeria, but something held me back from joining my more politically engaged colleagues in public protest.

"It was only when I won the Fields Medal in 1966 that my latent political and social convictions began to take an active form. In February of that year the Sinyavsky- Daniel trial took place, the first Soviet show trial of writers. They were given stiff sentences at labour camps. Along with the other medallists, I was to receive my medal at the meeting of the International Congress of Mathematicians in Moscow that summer. But I felt that I had to make some kind of protest against the Soviet authorities' persecution of Sinyavsky and Daniel, so I refused to travel there. My refusal attracted a certain amount of attention, and was inevitably criticized by orthodox communists.

"I had long been opposed to the (second) Vietnam war, and sympathized with the Vietnamese people for their suffering under the monstrously cruel American bombardment of their country. In 1967 I received a letter from a mathematician in Hanoi requesting literature about Algebra and Algebraic Geometry. This surprised me as I had been quite unaware that mathematical research was being pursued in North Vietnam, during the worst period of the Vietnam War. I sent as much material as I could. But then my new spirit of activism spurred me to travel to Hanoi myself to give lectures at the university there. After a week or so the cluster bombing by the US Air Force intensified so much that I and the members of my audience had to move to a hidden location in the forest some distance from Hanoi where I could continue my lectures. I returned to Paris deeply impressed with the spirit of the Vietnamese people and their calm confidence in the future, which has remained undimmed during their long struggle for independence and for the construction of a new society."

"Your experiences in North Vietnam," I observed, "must have had a profound impact on you."

"Yes, they did," Grothendieck affirmed. "They acted as a catalyst for my increasing involvement in pacifist, antinuclear, and environmentalist causes. Another stimulus was the student anti-establishment protests in France in May and June 1968. For many of the young, it was a time of political awakening, the moment at which questioning the established order was transformed into active protest against it. The newly radicalized younger generation came very close to unseating the establishment, in France anyway. The events brought the country to a revolutionary crisis. Starting with a student insurrection on one campus, the revolutionary fervour quickly spread through the student community, in the process surprisingly drawing in the working class, and culminating in a ten-million-strong general strike which brought the French government to the point of collapse. The country teetered on the very brink of revolution, and but for the inhibiting role played by the Communist Party and the trade union leadership, revolution might well have come about. What happened demonstrated the possibility, at least, of achieving that elusive goal, revolution in advanced capitalist countries—a goal which now seems impossible of realization.

"I was shocked to find that some of the radicalized students regarded me as a member of the establishment. But on reflection I realized that their point was just. While not a member of the establishment in spirit, as a professor at the IHES, drawing a substantial salary, I was very much a member in practice. This realization further catalyzed the transformation of my outlook. I had come to see that the military-industrial excesses of contemporary society, with its proliferation of nuclear weapons and unchecked economic expansion, would lead in the end to the destruction of the natural world and human society along with it. My critique was directed in particular at science, which I saw as playing a central role in advancing the military -industrial agenda, and had generated a new religion which I called "scientism". Especially poignant for me was the fact that I had come to see mathematicians as functioning, if unconsciously, as the religion's priestly caste, with mathematics itself serving as a kind of hieratic language. This naturally led me to question my professional mathematical activity.

Your critique of mathematics, " I remarked, "reminds me of Tolstoy's critique, on moral grounds, of the writing of novels, including his own."

"As a mathematician, or ex-mathematician, it would be disingenuous of me to claim that I have never thought of myself as at least comparable in ability with the mathematicians I sincerely admire. But the analogy you've made with Tolstoy, a great writer of genuine moral integrity, does surprise me, and, I have to admit, pleases me.

"But to continue retracing my own trajectory. In 1970 I was presented with the opportunity of testing whether my anti-establishment principles were sincerely held when I learned that the IHÉS, my place of employment for more than a decade, was receiving military funding. I felt impelled to resign my position there in protest.

Nevertheless, I still had a family to support, and, like everybody else except the independently wealthy, needed to work for a living. My prestige as a mathematician enabled me to obtain temporary employment at the *College de France* , but this didn't last long since I was unwilling to confine my lectures to mathematical topics. I was driven to expand my presentations to what I felt, and still feel, are the burning issues of the day, potential nuclear destruction, environmental degradation, the false god of science**.** In my mind these concerns had come to rival, and then to displace, my obsession with mathematics. I made a concerted effort to get others, in particular my mathematical colleagues, to join me in my campaign against the powers that be. "

'"Wasn't it around this time that you launched the *Survivre et Vivre* movement?"

"Yes, In July 1970," Grothendieck affirmed. "At that time I traveled to Montreal to take part in a summer school on algebraic geometry and commutative algebra. I'd been invited to speak on the esoteric topic of crystalline cohomology, but I was only prepared to accept the invitation under certain conditions. First, in addition to my mathematical lecture, I wanted to give a talk of the same length about the pressing ecological and other issues facing the world. And secondly, I insisted that this lecture should be announced in the same manner and with the same publicity as the mathematical one. Happily, the conference committee accepted these conditions without demur. So I gave a talk on these topics. In my talk I emphasized that scientists must wake up to the role they are playing in the drift towards the inevitable catastrophe of environmental collapse or nuclear annihilation. I made the point that we are sleepwalking towards Armageddon, ignoring our unique obligation to protect this wondrous world from the ravages of the powers of destruction unleashed by what we call the progress of science and technology. ''

"Yes, "I felt moved to interject, "The violence done by our species to the natural order."

''Precisely," Grothendieck continued. "But the point I was trying to make in my talk was that, for the last century or so, the most dangerous people in the world have been the scientists, not the generals or the ruthless political leaders who actually launch wars. And of course the reason for this is that the scientist, without hesitation, places in the hands of these powerful people the means of inflicting greater and greater violence on the natural order, as you have put it. In effect scientists, including, sadly, my fellow mathematicians, have become minions of power. I tried to make the point that the general lack of recognition on the part of the scientific community for its complicity in hastening the destruction of the planet was unconscionable, and that it was time to take action to combat it.

'"I was delighted with the enthusiastic response my oration received. The atmosphere in the lecture hall at its conclusion was so heady that the founding of the group *Survivre* followed spontaneously. "*Et Vivre*" was added sometime later, I can't recall exactly when. In August we issued our first bulletin, which contained a summary of the movement's goals. I don't recall the exact words we used, but they went something along the lines: 'We must fight for the survival of the human race and for all life, which is endangered by the pollution and destruction of the environment caused by industrial society and by military conflict'. One slogan we used at the time I recall with particular relish: 'The philosophy of unlimited growth is the philosophy of a cancer cell'."

"Brilliant!" I exclaimed. Then I went on: "After the *Survivre* movement was launched in Montreal, what kind of reception did it receive among French mathematicians?"

"Disappointing, with certain conspicuous exceptions," Grothendieck replied. "I was so sure of the validity of the goals of *Survivre* that, naively perhaps, I believed every rational person would come to share the point of view of *Survivre* once they became aware of the facts. Above all, I looked to my fellow mathematicians, whom I saw as being in the vanguard of rationality, to be the first to join the movement. Thus at the 1970 International Congress of Mathematicians in Nice I attempted to enlist my colleagues in the movement. I was greatly disappointed that my efforts were largely ignored, and in some cases actively rebuffed. Of course there were exceptions, such as Chevalley and Samuel, who joined the group and participated in the publication of the bulletin, which soon began to appear as *Survivre et Vivre.* And even though our group failed to expand in the way I had fondly envisaged, we compensated for our lack of numbers by redoubling our efforts. From 1970 to 1972 I was one of the principal activists in the group, to whose aims of cultural subversion I dedicated myself with an enthusiasm comparable to that which I formerly had for mathematics. As director and editor-in -chief of the monthly bulletins, I participated as much as possible in everything that happened in Paris, in the provinces, and also outside of France and especially in the United States where the "counter-culture" was in full swing. I must have spent six to eight hours every day on correspondence concerning our activities, and the rest of the time was dedicated to discussions, primarily meetings and conferences of our group. There were also activities directed towards the outside: public discussions on different topics - all concerning and related to the great crisis of civilization- in the ceremonial rooms of suburban town halls or in remote villages, in research institutes, universities, schools, in the most snobbish and the shabbiest localities, including a small suburban public school with its charming and somewhat bemused children. My professorial title and, on important occasions, my reputation as a scientific star served as an open-sesame with a certainty that never failed to astonish me. At no other time in my life had I dealt with so many people.

"Given the spirit of the times, other groups dedicated to the same goals as *Survivre* must have appeared on the scene. Did you have any contact with them?"

"We were aware of the formation of *Greenpeace, The Friends of the Earth* and S*cience for the People,* all of which shared our goals. A number of our members were in contact with these groups. Of particular interest to me was an invitation I received in July 1971 to participate in a meeting organized by a small group of mathematical logicians opposed to the funding of conferences in logic by NATO. There had been a number of previous protests by radical logicians against NATO involvement in logic which had attracted some support, but these protests were ignored by the authorities. When a NATO funded logic conference to be held in Cambridge, England in summer 1971 was announced, this group of radical logicians decided to sharpen their protest against NATO funding by organizing a "counter-conference" to be held in Denmark at the same time - August 1971 - as the official NATO funded one. The group secured the support of the Bertrand Russell foundation. Of course Russell himself - who had died the previous year - would have been totally opposed to NATO involvement in logic meetings. So the Bertrand Russell Memorial Logic Conference was born, and I enthusiastically accepted the organizers' invitation to participate in it. The meeting was held in a residential high school in Uldum, not far from Aarhus, Denmark's second city. The conference itself was an unusual mixture of mathematics and socio-political discussion, conducted in a very lively and informal atmosphere. The enthusiasm was infectious. I was only able to attend the first week of the meeting, but I was very busy. I gave a mathematical talk, and, more importantly, described my recent experiences in North Vietnam and initiated a group discussion on scientism. During this discussion I read out a preliminary draft of *The New Universal Church*, an editorial in a forthcoming issue of *Survivre et Vivre,* which provoked a lively response among the participants. At the end of the discussion one of the meeting's organizers offered to translate the essay into English and include it in the published conference proceedings. I readily agreed, and it duly appeared."

"It sounds like it was a lively affair," I observed, "very much a product of the *Zeitgeist."*

"Yes, it certainly was. I found the meeting, and the enthusiasm of the participants, most encouraging. Less encouraging, I am afraid, was my experience the following year at the summer school on modular forms in Antwerp. Like the Logic Conference in Cambridge, this meeting had been declared a NATO Advanced Study Institute. To add insult to injury, further funding was provided by capitalist entities such as Coca-Cola and Rank Xerox. Appalled by this, I appealed to the organizers and intended speakers to reconsider their participation. I circulated a letter to the organizers and expected participants of the meeting explaining my reasons for objecting to its sources of funding. I also staged an intervention at the meeting's opening event - which included the dispersal of red balloons for dramatic effect - and attempted to explain my point of view to the audience. But all I succeeded in doing was to drive the principal organizer of the meeting from the lecture theatre. Perhaps in my zeal to get through to the participants I overdid things. But, in any event, the resulting fiasco caused me to see clearly that the majority of my former colleagues could only think and act reasonably within the narrow domain of mathematics. They were blind to everything outside of it. I saw that I now had almost nothing in common with them."

"So you gave up trying to convince your fellow-mathematicians of the error of their ways and withdrew to the countryside."

"Yes, essentially," Grothendieck responded. "I was briefly a member of a commune I had formed with some kindred spirits, but after a while I found that communal life led to more personal entanglements than I felt comfortable with, so in 1973 I moved here, into comparative, although hardly total, solitude. I've found much contentment in the simple life I now lead, it's like living in a perpetual Sunday."

"Yes, it must be a relief!" I said. "I should tell you that I've read *The New Universal Church,* and was deeply impressed, even moved by it. If you're willing, I'd like to talk about 'scientism', from a philosophical point of view."

"Yes, let's do that."

"First of all," I continued, "it seems clear to me that scientism is a hypertrophied form of the ancient philosophical doctrine of atomism. This is the idea, going back in the West to the atomists Democritus and Leucippus that there is nothing in the world but material atoms moving and interacting in a void. This materialist conception expanded into the doctrine that all that truly exists- Objective Reality - is material, and 'minds', 'souls', ‘emotions’ , ‘consciousness’ - in fact the whole realm of the Subjective - is an illusion. Practical, and later theoretical science evolved on this identification of objective reality with matter. This form of materialism has come to came to be regarded as ‘progressive’ - in the West at least - because of its opposition to traditional religions. It is indeed ironic that it has evolved, as ‘scientism’, into a new religion itself, in which chance has replaced purpose, and from which the Subjective has been banished along with God, the soul and the rest of the components of traditional theology. This credo is given remarkably passionate expression, considering the bleakness of its message, by Jacques Monod - one of your colleagues at the *College de France*, I believe - in the conclusion of his book *Chance and Necessity:* ‘Man knows at last that he is alone in the unfeeling immensity of the universe from which he emerged by chance. His destiny is nowhere spelled out, nor is his duty. The kingdom above or the darkness below – it is for him to choose.’ Taken out of context, one could be excused for thinking that this passage had been written by an existentialist such as Camus or Sartre - it certainly doesn’t reek of scientism! Of course, existentialism itself could well be viewed as a kind of rearguard action taken against materialism by philosophical idealists.”

At this point Grothendieck broke in: “I didn’t get to know Monod when I was at the *College de France.* I was too busy engaging in cultural subversion, which I doubt would have met with Monod’s approval. I respected him as a stalwart of the Resistance during the German occupation, and as a man of great ability. But in my eyes he had become a pillar of the scientific establishment, and hardly a potential recruit for *Survivre!* I read his book, and, while admiring the elegance of his prose, I thought it almost a perfect bible of scientism. As you point out, the book emits more than just a whiff of existentialism, and this is hardly surprising since Monod and Camus were close friends. But, unlike Monod, Camus was no adherent of scientism. It may well be the case that he wouldn’t have become one even if he had been subjected, as Monod was, to the *deformation professionnelle* experienced by the majority of scientists.”

“Yes, I agree,” I said. “Camus and Monod may well have shared the view that existence is purposeless, and a product of chance but, unlike Monod, Camus did not require that insight to be authenticated by the quasireligious dogma of scientism.”

“ Precisely!” Grothendieck declared. “As it happens, I don’t believe that existence is purposeless, but even if I did, I would, along with Camus, regard that as an insight not requiring the dubious blessings of scientism to confirm it.”

“If you’re amenable,” I returned, “I’d like to pick up the problem of science and technology and their potential for ecological destruction.”

‘’By all means.”

“It seems to me,“ I continued, “that there is an essential difference between nuclear weapons with their capacity to sterilize the earth’s surface completely, and other technological developments which have had dire, if less destructive consequences. I've often heard it said that the creation of nuclear weapons was ‘inevitable’. But the historical record doesn't in my view bear this out. The initial development of the atomic bomb was triggered by very specific historical circumstances, namely the fear of the Western Allies in World War II that Nazi Germany might obtain it. Mercifully, if ironically, the Germans never succeeded in developing the atomic bomb, apparently believing that carrying out the task was technically impossible. While it is true that physicists in the 1930s saw the theoretical possibility of developing an atomic bomb, they also saw that there would be virtually insuperable difficulties in actually constructing one. The creation of nuclear weapons was the result of a unique historical situation. Had Nazi Germany not emerged, it seems rather unlikely that any state would have felt it necessary to make the colossal effort needed to produce them. Of course, now that the genie has been released from the bottle and nuclear weapons are almost commonplace, it is easy to regard their creation as ‘inevitable’ and to forget just how special the historical conditions leading to their emergence actually were. While nuclear weapons are unfortunately still with us, it seems to me that, from a purely political point of view, it would be comparatively easy, in principle at least, to eliminate them. No state wants them for intrinsic purposes. Indeed, apart from their first use on Japan by the US at the end of World War II, they have (so far, and luckily) functioned solely as a deterrent to attack by other states, and they are widely acknowledged as being virtually useless for any other purpose. It is not inconceivable that political negotiation between states could lead to their elimination: this would only require the agreement of a handful of heads of state and their advisers. Political negotiation has in fact already begun at least to limit their production. Their complete elimination is by no means impossible.

“Other technological and economic developments which have had, or will have, environmentally destructive consequences have a somewhat different character. I agree that it is reasonable to equate the philosophy of unlimited growth with the philosophy of a cancer cell, but it is after all in the nature of a cancer cell to consume its host. It’s at least possible that unlimited growth might not actually be an intrinsic part of human nature.

Still, it's hard to deny that it is a direct consequence of the natural acquisitiveness of human beings.

"Here's another example. In the late nineteenth century the Swedish chemist Arrhenius made the striking prediction that if the amount of carbon dioxide in the atmosphere were to be doubled, then the earth's temperature would rise by 5 to 6 degrees Celsius. If this 'global heating' were actually to occur, the effect on the environment would be catastrophic, with rising sea levels, loss of millions of animal species, and many other horrors. Now the main source of increase in carbon dioxide emission into the atmosphere is the burning by human beings of fossil fuels in energy production. In the 20th century, with the unprecedented expansion of oil and natural gas production, the rate of emission of carbon dioxide has reached disturbingly high levels, and it seems increasingly likely that, unless the burning of fossil fuels is drastically reduced, Arrhenius' scenario, with all its dire consequences, will become a reality. Now if we grant that the burning of fossil fuels is an extension of the 'natural' human activity of burning wood for the purposes of heating or cooking, then you might reasonably claim that global heating, if it does occur, was 'inevitable'. And, given that very 'naturalness' of the human activity causing the global heating (to say nothing of the skepticism many people might have about its origin), it would seem to be very difficult in principle to get people to agree to do something about it. Unlike the case of nuclear weapons, which could in principle be eliminated by the collective decision of a few heads of state, modifying the human activity giving rise to global heating would require radical changes in the lifestyle of virtually every person on the planet. And most people would be unwilling to countenance, let alone implement, those changes. Everybody- aside from a few fanatics - would be happy to see the end of nuclear weapons. But even if people became generally convinced that global heating is caused by human activity, it would be only with great, and sadly, natural reluctance that they would abandon their cars and the other trappings of modern civilization which are dependent on the burning of fossil fuels. "

"Yes, that's a good point," Grothendieck said after a pause, "It's true that the creation of nuclear weapons is the ultimate madness, but at the time it seemed more than possible that the psychopathic nature of the Nazi regime would lead it to develop and use the weapons. Yet in the end it was an allegedly 'rational' regime, the United States, that produced that triumph of physics, the atomic bomb, and used it on Japan. I summed this up in the equation 'Physics equals Hiroshima'. But I do agree that fundamentally only the political power of a small number of individuals stands in the way of eliminating nuclear weapons. In the case of the more than possible 'global heating' produced by the burning of fossil fuels, I agree that it would be more difficult in principle to curb it, because billions of people have become dependent on technologies -such as the generation of electricity and the propulsion of automobiles - which are powered most easily by fossil fuel combustion. On the other hand, although global heating would indeed be dire, and make life on earth more difficult, it probably would not eliminate life altogether, as would be the case if nuclear weapons were used without restraint. But of course both are monstrous violations of the natural order, driven by the worship of science."

" Physics equals Hiroshima!', I exclaimed. "Poor Einstein! He was an honourable man, and I don't think he ever overcame his guilt feelings at his involvement, however indirect, in the creation of the atomic bomb. I'd very much enjoy discussing this with you further. But our time together is short and I'd like to use what remains in discussing the nature of mathematics and the creative process underlying its creation. Is this agreeable?"

"Sure," replied Grothendieck with a smile. "Let's do that."

"Well, first of all, do you regard your work, and mathematics generally, as being primarily the product of discovery, or of invention? I'm inclined to think that in the mathematician's activity there's a kind of dialectic between discovery and invention. First one discovers the capacity for invention in oneself, and then, once one invents, one discovers hidden properties of those inventions which were not consciously invented."

"I think you've put your finger on an essential aspect of the psychological conditions under which mathematics is produced by working mathematicians. But let me respond to your original question. It's a long-standing tradition to distinguish three kinds of 'qualities' or 'aspects' of things in the Universe which are amenable to mathematical reflections: Number, Magnitude and Form. Of these it is Form or Structure that has the greatest fascination for me. And, among the numerous ways that Form manifests itself, the one that fascinates me above all is the structure concealed within mathematical objects. Once a mathematical object is grasped by the mind, its hidden structure is immanent, it cannot be 'invented', but only 'discovered' in the near-literal sense of being revealed. The most we can do is to attempt to bring this indwelling structure to light with patience and humility. If there is some sort of inventiveness in this work, we aren't in any sense 'making' or 'building' these structures. They hardly waited for us to find them in order to exist, exactly as they are! We didn't 'invent' them. But we are, so to speak, 'summoning them from the Void', that is, asking them to take a definite shape within our own consciousness. In order for this to happen, we are constantly driven to invent the language most appropriate to express, with increasing refinement, that indwelling structure of mathematical objects. The sole thing that constitutes the true 'inventiveness' and imagination of the researcher is the quality of his attention as he listens to the voices of things. Nothing in the Universe speaks on its own or reveals itself just because someone is listening to it. One must do more than listen, one must hear."

"From your eloquent response," I said, "I infer that you cleave to the view that mathematical Form is discovered, and not invented by us. Thus I take it that you would essentially agree with G. H. Hardy, who wrote 'I believe that mathematical reality lies outside us, that our function is to discover or observe it, and that the theorems which we prove, and which we describe grandiloquently as our "creations," are simply our notes of our observations. This view has been held, in one form or another, by many philosophers of high reputation from Plato onwards.' Would you agree with him?"

"I agree to the extent that I believe mathematical form to be part of objective reality," rejoined Grothendieck. "At one time I probably would have agreed with Hardy's characterization of the mathematician as a kind of 'discoverer' or 'explorer' along the lines of Christopher Columbus. But I have come to see the truly creative mathematician as a kind of inspired architect, who, in designing a building, uses his imagination to bring out structure that is already potentially 'there'. Of course the architect wishes his design to be realized as an actual building, while in pursuing his inspiration the mathematician's purpose is to create a kind of hall of mirrors faithfully reflecting the structure and beauty concealed within the mathematical objects to which he has directed his attention. "

"Over the last two or three centuries," Grothendieck continued, "the natural sciences, and above all, mathematics, have gradually liberated themselves from the religious and metaphysical assumptions of their culture and time, which have acted as brakes on the universal development (for better or worse) of a scientific understanding of the universe. Nevertheless it's true that some of the most basic and fundamental notions in mathematics, such as spatial translation, the group, the number zero, the techniques of the calculus, the designation of coordinates for a point in space, the notion of a set, of a topology, let alone negative and complex numbers, required millennia for their emergence and acceptance. These may be considered so many eloquent signs of that inherent 'block', implanted in the human psyche, against the conceptualization of totally new ideas, even when these ideas possess an almost infantile simplicity, and which one would think would be obvious based on the available evidence, over many generations.

In this connection one recalls particularly the incomprehension that Galois' work met with initially."

"You've saId that mathematicians, and presumably scientists generally, should show patience and humility in their work. That certainly doesn't seem to be the case today."

"You're right. In our age the frenzied productivity in the domain of science is comparable with that of weapons and the consumption of material goods. Yet at the same time it is a long way from the dynamism and independent spirit of our 17th century predecessors. They didn't wait to receive external support before developing the infinitesimal calculus, they just plunged ahead, not worrying whether what they were doing was rigorous or pure conjecture. Nor did they wait for some eminence among them to give them the green light. They were not afraid to grasp and work with that which everybody could see first-hand with their own eyes."

"Many of your own outstanding mathematical achievements are, unfortunately, beyond my grasp,"I said, "but I am reasonably familiar with one of your most important concepts, that of a topos. This concept, which you introduced into algebraic geometry at the beginning of the 1960s, has proved to be of amazing fertility, uniting topology, algebra, set theory and logic. Would you like to say something about it?"

"Ah yes, the topos!" Grothendieck responded. "The idea of mine I cherish above all. I see the topos concept as a kind of universal residence for the vast new geometry that has appeared in the past few decades. It offers a glorious fusion of geometry, topology and arithmetic, mathematical logic, the theory of categories, and that of continuous and discontinuous or 'discrete' structures. I regard it as my grandest conception, devised in order to grasp with precision, in the same language rich in resonances of geometry, an 'essence' common to the most disparate situations, coming from every region of the universe of mathematical objects."

"The topos concept is a generalization of the concept of sheaf, is it not?"

"Yes," resplied Grothendieck, "but more precisely, it generalizes the notion of a category of sheaves on a topological space. A sheaf on a given topological space is determined by the ensemble of open sets of the space, together with the 'covering' relation between an open set and families of open sets covering it. The ensemble of open sets of the space forms a (small) category, and I saw how to extend the idea of coverings by open sets to ‘coverings’ of objects by maps in an arbitrary small category. A small category equipped with a system of such ‘coverings’ of objects (later known as a Grothendieck topology’) I called a ‘site’. It was then easy to extend the notion of sheaf on a topological space to sheaf on a site, giving rise to the category of sheaves on a site. The term ’topos’ was then given to any category equivalent to the category of sheaves on a site: such a category can be thought of as a generalized ‘space’. My students Verdier and Giraud both played important roles in the emergence of these ideas. In particular Giraud formulated a set of purely category-theoretic conditions characterizing a topos. In the late 60s Lawvere and Tierney came up with an elementary axiomatic description of toposes. This was based on Lawvere’s insight– which had not occurred to me - that every topos has a ‘subobject classifier’, namely, a single object Omega having the property that subobjects of any given object are naturally correlated with maps from that object to Omega. Omega can naturally thought of as an object of ‘truth values’, which brings logic into the picture. Lawvere had also seen that, in any topos, the underlying site can be represented by certain self-maps on Omega satisfying what logicians call ‘modalities’. A category satisfying the axioms of Lawvere and Tierney was quickly christened an ‘elementary topos’ and, as their near-ubiquity becomes clear, will soon, I predict, simply be called a ‘topos’. It has emerged that toposes in this sense can be thought of not only as generalized ‘spaces’, but also as generalized ‘universes of sets’, and generalized ‘theories’, each with its own ‘internal logic’.”

“It’s especially remarkable,” I observed, “that Paul Cohen’s construction of a model of set theory in which the continuum hypothesis is false, thereby establishing its independence of the axioms of set theory, can be seen as an instance of the construction of a sheaf topos. And that his concept of ‘forcing] of statements follows the ‘internal logic’ of the topos.”

Taking up the thread, Grothendieck said, “Cohen managed to solve an outstanding problem in set theory ‘from scratch’ by introducing forcing, a general method for constructing models of set theory with many different properties. Once it was realized that a topos can be seen as a ‘generalized model of set theory’, it became natural to suppose that methods of constructing toposes could be used to construct models of set theory. Of course, this observation is in no way intended to belittle Cohen’s achievement, which was made entirely independently of the topos concept (itself only just emerging at the time Cohen was doing his work), and which was the result of deep insight on his part. When we received our Fields Medals in 1966, neither of us had the slightest inkling that there was any connection between our works! It was in fact Bill Lawvere who later pointed out the connection."

"I'd like to ask you one final question, on a different topic. It has often been remarked that mathematicians are fond of music. Is this true in your case and, if so, could you name your favourite piece?"

"Yes, I enjoy classical music, especially Bach. My favourite works of his are the sonatas and partitas for solo violin, in the recordings by Jascha Heifetz."

I was delighted by Grothendieck's response, and I felt that it offered an agreeable note on which to end our discussion. I got to my feet and said, "How wonderful! Heifetz playing Bach is my favourite also. Incomparable music played by an incomparable violinist. On that note, if you'll excuse the pun, I'll now take my leave. You have been most gracious in talking to me at such length, and I've found our discussion most fascinating. I thank you very much. "

Grothendieck rose, and as we shook hands, said: "I too have enjoyed our discussion. I wish you well."

I extracted from his jacket pocket the sealed envelope I had prepared and handed it to Grothendieck. "This is for you. I believe you'll find the contents interesting. Farewell." Then I turned, and quickly departed.

Aldous Huxley

My researches into 20th century literature had quickly led me to the novels and philosophical writings of Aldous Huxley. I happened to be acquainted with his father Leonard during his time as an editor of the *Cornhill Magazine* at the beginning of the century. A polymath of extraordinary range, Aldous Huxley was one of the most brilliant intellects of the 20th century. I became fascinated with his evolution from an ultra-sophisticated young literary gadfly into the intellectual and spiritual guru of his later years. Huxley's literary reputation was first established in the years immediately following the First World War. His early novels such as *Crome Yellow* and *Antic Hay* were dazzlingly witty satires on the pretensions of the English leisured intellectual class in which he had himself grown up. In his *Point Counter Point* of 1928, on the surface a novel of manners written in his characteristically brilliant style, his characters wrestle, through focussed conversation, with the philosophical and ethical problems which had preoccupied him since his youth. It is in *Brave New World* (1931), justly his most popular book, that Huxley's concern for the future of humanity is first made explicit. Cast in the form of a tragicomedy, the novel describes a totalitarian, hierarchical society of the future based on wholesale genetic engineering. While *Brave New World* was intended as a satire on the utopias of H. G. Wells, its apparently antitotalitarian message established Huxley as an icon of the liberal tradition.

In his later years Huxley's developing spirituality began to assume a concrete literary form. In *The Perennial Philosophy* (1945) he presents an anthology of excerpts and commentaries designed to illustrate what he called "the highest common factor of all the higher religions." In the 1950s Huxley experimented with altered states of consciousness through the use of psychotropic drugs such as mescalin, as reported in *The Doors of Perception* }(1954) and *Heaven and Hell* (1956).

Huxley died on 22 November 1963, the same day on which US president John F. Kennedy was assassinated.

I came to see Huxley's life as a journey from intellectual elitism to a profound spirituality. I was eager to meet and talk with him.

I needed to choose a propitious moment for my meeting with Huxley. From my biographical investigations I had learned that during his tenure in Fall 1960 as a Visiting Professor at the Massachusetts Institute of Technology he had not merely given lectures, but had made himself freely available for discussion with his students. Surely an opportune time for our tête-à-tête, I thought. I decided to pose as a staff member of *Vogue* magazine.

In preparation for our meeting I time-travelled to 1969 Oxford, where at Blackwell's famous bookshop I purchased a copy of the recently published *The Mind of a Mnemonist: A Little Book About a Vast Memory,* by the eminent Russian psychologist A. R. Luria. I intended to send this book to Huxley after our meeting. I was sure that he would be fascinated by Luria's account of a patient possessing both unbounded eidetic recall and acute synesthesia under which he would "see" sounds and "taste" feelings, in a way very similar to the experiences Huxley himself had undergone in his experiments with hallucinogenic drugs. Of course I also knew that Huxley could not have read the book, since it was not published until several years after his death. In giving him a book published in the future, I was hinting at my own status as a chrononaut. As with my parting gift to Grothendieck, I convinced myself that this would amount to no more than a harmless breach of my self-imposed chrononaut's code.

I time-travelled to Boston in October 1960 and took a room in a hotel there. I planned to attend Huxley's scheduled lecture at MIT on October 26th and approach him after its conclusion. The lecture itself, part of the series *What a Piece of Work is a Man,* which took place in a crowded lecture theatre on the MIT campus, was most impressive both in its range of ideas and mode of delivery. Huxley's gentle, cultured voice and manner charmed his audience. At the end of his lecture he was bombarded with questions which he answered with thoughtfulness and grace. As the audience dispersed I went up to him and introduced myself as a staff member of *Vogue* seeking an interview. He told me that he would be happy to meet with me at his apartment at 2 p.m. on the following Tuesday.

At the appointed time I arrived at Huxley's apartment and knocked on the door. It was opened by a woman of striking appearance whom I recognized from photographs as his second wife Laura. She looked me up and down and asked me, in a pleasing Italian accent, if I was "the man from *Vogue* come to interview my husband." When I confirmed this she smiled and invited me inside. She conducted me into the living room, where Huxley was seated in an armchair. As I entered he got up—very tall and thin, it was as if he unfolded himself from his chair—and greeted me. "Mr. Bellane? I'm pleased to see you. Do take a seat." He gestured me toward the armchair opposite his.

“Mr. Huxley,“ I said after I had sat down, ”I’m flattered that you remember my name after our very brief encounter the other day. And I’m delighted that you’re granting me the privilege of conducting an interview with you. Now while it is my intention to put some provocative questions to you, I have every hope that our tête-à-tête will prove to be not merely a dry formal interview, but an exchange of ideas perhaps worthy of comparison with the conversations of the various characters in your early novels. “

“Your proposal, Mr. Bellane, intrigues me,” Huxley responded with a smile. “Do start the ball rolling.”

I suddenly realized that I had forgotten to activate my recorder. Reaching surreptitiously into the side pocket of my jacket, I switched it on.

“Well, Mr. Huxley, let me begin by observing that, while your writings display a dazzling breadth of erudition, there is one theme that threads its way through much of your work. This is the fact that the abilities, and so also the attainments of human beings differ widely, a fact that you hold has profound consequences for the organization of human society. Would you consider this a fair assertion?”

Huxley paused for a moment and then replied with gracious equanimity. “Yes, eminently fair. I have always been concerned with the problem of the distribution of human abilities. As a boy I aspired to follow my brother Julian into biology but was thwarted by eye disease. Had I been able to realize my ambitions, I would surely have actively pursued my early interest in how differences among individual members of a species drive its evolutionary development. It is an analogous interest in the individual differences among human beings, and their origins, that you have correctly identified as an abiding theme in my work. It was this interest that led me, as a young man, to the belief, which I shared with many intellectuals of the time, that hierarchical government by some sort of politico-cultural elite offered the only way out of the chaos to which Western society had been brought by the Great Depression. The horrors of Hitler’s Germany and Stalin’s Russia disabused me of that view.

“Were you not also a supporter of the Eugenics Movement?” I enquired.

“Yes,”Huxley responded, “I have to admit I was. This was fashionable among intellectuals of the day - you may know that my brother Julian has for many years been a prominent member of the British Eugenics Society. Eugenicists held the view - and I then subscribed to it - that the human species could be improved by 'breeding out' disease, disabilities and other undesirable hereditary characteristics from the human population. While this seemed an admirable idea in principle, it was, as I began to see, tainted by an overestimation of the influence of heredity, and a corresponding underestimation of environmental factors, in the development of human beings. For example, eugenicists were initially convinced of the hereditary origin of a number of less than desirable human attributes such as certain types of mental illness, criminal tendencies, and even, absurdly, poverty. It later came to be accepted that many of these unwelcome attributes were environmental, rather than hereditary, in origin. I came to see that is no valid basis for estimating and comparing the intrinsic worth of different individuals, without the presence of economic and social conditions which provide approximately equal opportunities for all members of society instead of stratifying them from birth into classes with widely different privileges. "

"Had you come to hold this view by the time *Brave New World* was written?"

"Well, " Huxley responded with a smile, "the idea may have been no more than inchoate in my mind at that time. But in *Brave New World* I intended, *inter alia,* to cast a satirical, if fanciful eye on the idea of a society whose *raison d'être* is class stratification through the imposition of profoundly *unequal* conditions under which its individual members are born. I depicted, with a certain ribaldry, a society in which stability has been achieved by imposing excessive order through technological means. The result is a society of contented termites. I came to believe, and still believe, that humanity should struggle to prevent a society resembling that of *Brave New World* from becoming a reality. I am sorry to say that the prophecies I made in *Brave New World* are coming true much sooner than I thought they would. The blessed interval between too little order and the nightmare of too much has not begun and shows no sign of beginning."

I nodded in agreement, and said, "*Brave New World* is a society in which inequality and social stratification has been created through the actual manufacturing of individuals with intrinsically superior and inferior abilities. But what if social conditions were such as to provide approximately equal opportunities for all members of society? Would the natural distribution of abilities among individuals still result, willy-nilly, in a stratified society?"

"It seems likely, unfortunately," Huxley responded. "An excellent illustration of the tendency for social classes to crystallize is provided by Michael Young In his recently published book, *The Rise of the Meritocracy*. There he describes a future society in which social stratification is the result of judging people entirely on what their intrinsic abilities are taken to be. Aristocracy of birth has turned into an aristocracy of talent. The author's message is that, while it makes good sense to appoint individual people to jobs on their merit, it is quite the opposite when those who are judged to have merit of a particular kind harden into a new social class without room in it for others. "

"What do you think is the source of the natural distribution of abilitiesamong human beings?" I ventured to ask.

"Well, Huxley resoinded. "The sources of this distribution have been hotly debated. I think that few nowadays would deny that, in a given population, if the conditions under which each of its members are born and raised are identical (of course in practice this is never the case, but I am speaking only hypothetically), then differences in attributes among its members must be traceable to differences in their genotypes. In other words, if the environment is fixed, then genotype determines phenotype. This relationship - let me call it 'Mendel's law' after that pioneer geneticist- superficially resembles Boyle's law for gases, that the volume of a given mass of a gas at a constant temperature is inversely proportional to - and so determined by - its pressure. But there are important differences between the two. In the case of Boyle's law the meaning of the relevant terms - volume, temperature, pressure - are precisely defined and quantifiable. But in 'Mendel's law' this is far from being the case. Here the key terms 'environment', 'phenotype', 'genotype' are by no means precisely defined in the scientific sense. Nevertheless, the first two terms have a clear everyday meaning. When Mendel pursued his investigations into the heritability traits of pea plants, it was clear to him what was meant by the 'environment' in which his plants grew. The term 'phenotype' had yet to be coined, but in selecting for study a bundle of traits such as shape, flower color, pod shape, etc., he effectively supplied a concrete meaning, *avant la lettre,* to the term. The word 'genotype' had also not yet been invented, but Mendel introduced a term - 'elementen' - which is a forerunner. Mendel conceived the 'elementen' of an individual plant as an entirely abstract, unobservable entity, a kind of 'essence' postulated to exist within the plant which determines its phenotype but, unlike the latter, is transmissible according to certain rules from a plant to its seedlings. With the rise of molecular biology and the sharpening of insight into the mechanism of biological inheritance, Mendel's 'elementen' became identified as genes, certain assemblages of macromolecules with a definite, if complex physical structure. The collection of an individual organism's genes- its genotype - thus became an objectively existing physical entity. But the notion of phenotype remained somewhat ill-defined. What 'traits' should be included in the phenotypes of higher animals, human beings in particular? It seems obvious that simple, physical attributes such as height or eye colour should be part of the phenotype, but what about complex abilities such as the capacity to use language, to play the piano, to master higher mathematics, or indeed to be 'intelligent' in general? It seems unproblematic to affirm 'Mendel's law' for phenotypes composed solely of simple attributes. But the situation is less clear when phenotypes are expanded to include complex abilities such as the ones I have just mentioned. These abilities can only be manifested in the presence of certain cultural and social conditions - without pianos there would be no pianists, precocious or otherwise! Nevertheless it might seem reasonable to extend 'Mendel's law' to expanded phenotypes -we are, after all, inclined to regard the exceptional attainments of child prodigies such as Mozart or Gauss as being mainly the result of 'inborn' abilities, since in their case environmental influences have not had time to have a significant effect. But in accepting that genotype determines such enlarged, more complex phenotypes, we have to recognize that genotypes themselves, along with their patterns of heritability, must necessarily become more complex. This makes the identification of heritability much more difficult. This is particularly true of intelligence. Even when intelligence is reduced to the quantifiable - and doubtless inadequate - form of IQ, the element of heritability is extremely difficult to disentangle from the environmental conditions under which individuals develop. This issue, which is a vexed one, has played an important, and controversial role in the formulation of educational policy in a number of countries."

It seems to me," I said ,"that while differences among individuals do have a genetic basis, it is mistaken to claim further that differences among socially defined groups likewise have a genetic basis. And the idea that genes determine the fate of an individual is surely false as well. Medical predictability can rarely be based on genetic analysis alone; environment also plays an important role. Something as complex as intelligence is likely to be affected by many factors beyond an individual's genes. "

"Yes, I wholeheartedly agree," Huxley responded quickly. "The claim that differences among socially defined groups have a genetic basis is an entirely unwarranted, even pernicious, extension of Mendel's law. An especially poisonous instance of this claim was the justification of black slavery on the grounds that black people are genetically inferior to whites. As for the genotype itself, I see it as determining a person's *uniqueness*. But just as one's *dharma* does not determine one's *karma,* one's genotype does not determine one's fate."

"*Brave New World* has often been compared with Orwell's *1984,"* I said. "But while the societies portrayed in both books are totalitarian , they differ profoundly in their ultimate goals and the means by which these goals are to be reached. In *Brave New World* the goal is the creation of a global society in which stability has been established by means of chemical and psychological conditioning. In the society of *1984* the goals and the means of achieving them are quite different from those in your novel. In *1984* the goal of the ruling oligarchy is the attainment of total power over the minds and bodies of human beings using brutal coercion. This is attested toby the chilling declaration of O'Brien, the Party's chief interrogator: *The Party seeks power entirely for its own sake. We are not interested in the good of others; we are interested solely in power. Not wealth or luxury or long life or happiness: only power, pure power. Power is not a means, it is an end. The object of persecution is persecution. The object of torture is torture. The object of power is power. If you want a picture of the future, imagine a boot stamping on a human face—forever.* The explicit sadism expressed in this declaration is light-years away from the comparatively benign intentions of the ruling class you portray in *Brave New World."*

At this point I paused, and looked at Huxley with an expression I hoped would invite him to pick up the conversation.

Huxley nodded gently, and allowed a few moments to pass before responding: "Of course you are right in recognizing the profound differences in the societies delineated in our two novels, even though both were intended to embody forms of social stability. I hope Orwell's shade will forgive me for my presumption here! I presented the society of *Brave New World* as one in which its motto COMMUNITY, IDENTITY, STABILITY has been fully realized, where each member knows his or her place, and is, with very few exceptions, content with it. This has been achieved by chemically conditioning human embryos and psychologically conditioning the resulting infants so as to produce adults who are happy with their lots. Members of the lower social castes are happy because they are totally unaware that their happiness has been conditioned, and members of the upper castes, while they are aware that they have been so conditioned, happily accept the fact, and their social status, because their conditioning has been designed precisely with that acceptance in mind. I recall that one of the Betas declares: '*Alpha children wear grey. They work much harder than we do, because they're so frightfully clever. I'm really awfully glad I'm a Beta, because I don't work so hard. And then we are much better than the Gammas and Deltas. Gammas are stupid. They all wear green, and Delta children wear khaki. Oh no, I don't want to play with Delta children. And Epsilons are still worse.'* The conditioning is reinforced for adults of all classes through high-tech entertainments, and the use of an ideal drug intended to give pleasure, numb pain, and prevent any questioning of the status quo. In

Brave New World conformity is achieved entirely by conditioning and pleasurable distraction.

"By contrast, the society Orwell described in *1984* is controlled by its rulers, the Inner Party, largely through punishment and the fear of punishment. The party's official ideology, Ingsoc, contains certain subtler elements, parodies of the debased Stalinist version of Marxist dialectic, with its glorification of contradiction, which Orwell abominated. You'll recall Ingsoc's defining slogans WAR IS PEACE, FREEDOM IS SLAVERY, IGNORANCE IS STRENGTH, and the names of the four ministries through which the Party imposes its rule: the Ministry of Peace whose concern is war; the Ministry of Plenty which deals with scarcities; the Ministry of Truth, which handles propaganda and in which history is rewritten; and the Ministry of Love—headquarters of the dreaded Thought Police—which dispenses law and order. You'll also recall that the Party's intellectuals are apparently hard at work on the development of 'Newspeak', a debased version of the English language designed to make the expression, even the very holding of unorthodox opinions impossible. Ultimately it was hoped to make articulate speech issue from the larynx without involving the higher brain centres at all. The core vocabulary of Newspeak has been reduced to such mind-numbing words as 'plusgood', 'bellyfeel', 'plusgood' and 'doublethink', a term fully encapsulating the debased dialectic of Ingsoc. You seem to have an excellent memory. Perhaps you can recall how Orwell defined doublethink…"

Huxley paused, and looked at me as if, now, he wanted *me* to complete *his* thought. Fortunately Orwell's definition of "doublethink" was one of my favourite passages in *1984* and I was able to quote it from memory.

" If I recall correctly, it was: *To know and not to know, to be conscious of complete truthfulness while telling carefully- constructed lies, to hold simultaneously two opinions which cancelled out, knowing them to be contradictory and believing in both of them; to use logic against logic, to repudiate morality while laying claim to it, to believe that democracy was impossible and that the Party was the guardian of democracy; to forget whatever it was necessary to forget, then to draw it back into memory again at the moment it was needed, and then promptly to forget it again: and, above all, to apply the same process to the process itself."*

"Bravo! Your memory is indeed excellent," Huxley exclaimed. "Yes, Orwell's description of the psychological state that the Party authorities wished to induce in its subjects is nothing short of brilliant. But I have long thought that the crude sadistic methods of *1984* would prove most ineffective in bringing about these rather subtle psychological states.

It would amount to using a sledgehammer to crack a nut, with the likely result of obliterating the nut altogether. Far more effective in achieving the desired result, it seems to me, would be the infant conditioning and narco-hypnosis of *Brave New World* . "

"I agree," I said. "But then the authorities of *1984* are interested not in subtlety, but only in the actual practice of domination. This poses a risk for Party intellectuals such as O'Brien who truly understand the nature of Ingsoc. O'Brien has grasped that Ingsoc is a kind of collective solipsism. There is nothing at its core but the will of the Party and its unbounded capacity for inflicting pain. No objective reality exists into which one can escape. But in accepting the actual truth of these assertions O'Brien is placing them, in his own mind, beyond the applicability of doublethink, which, as he knows, is incompatible with the concept of unadorned truth. If he truly accepts the principles of Ingsoc, he must, like a genuinely devout Catholic priest who has lost his faith, turn himself in to the authorities, leading to his own destruction."

"Yes, exactly," said Huxley." You have put your finger on the self-contradiction at the core of Orwell's vision of a future society. This is bound to become a source of disturbance for its intelligentsia, and thereby a potential source of instability. In the imaginary world of my own fable sources of instability will no doubt exist, as they do in every society. But it is highly unlikely that instability would arise as the result of dialectical subtleties causing disquiet in the intelligentsia of its ruling caste. There are no inherent contradictions in the ideology underlying the social order I portrayed in *Brave New World*. The worst thing that might disturb the more unsettled minds among its higher castes and weaken their faith in happiness as the Sovereign Good would not be the shock of actual contradiction. Rather, it would be a vague feeling of dissatisfaction with the confines of the present human sphere, a feeling that that the goal of human life was somewhere beyond, that its purpose was not merely the maintenance of well-being, but some intensification and refining of consciousness, some enlargement of knowledge.

"In fact," I observed, "in your novel the problems faced by intellectuals are comparatively trivial."

"Yes, certainly," Huxley responded . "But there will inevitably be people, not necessarily intellectuals, who, for one reason or another, have got too self-consciously individual to fit into community life, who aren't satisfied with orthodoxy, who've got independent ideas of their own. Their punishment, if it can be called such, is to be, like Napoleon, sent to an island. But not into solitude, rather to a place where they will meet the most interesting set of men and women to be found anywhere in the world, people who have original ideas of their own. The nearly perfect control exercised by the government of *Brave New World* has rendered punishment largely unnecessary. When punishment is inflicted, it is mild. Government control is hardly even noticed by the populace, since its is achieved by subliminal conditioning and continual reinforcement of desirable behaviour. The essential difference between the society of *Brave New World* and that of *1984 is* that the former is based on the reception of pleasure, and that of *1984* on the inflicting of pain. My novel was intended to present, in a satirical, even light-hearted manner, a stratified, yet trivialized Heaven, with all its superficial pleasures, realized through a seemingly rational technology. Orwell's novel, by contrast, presents a profoundly fearful, irrational version of Hell. Orwell thought that the fear of pain is deeper in us than the anticipation, maybe even than the actual experience, of pleasure. He may well have been right. At any rate, it is this visceral conviction that is the source of the remarkable power of his novel. But as for providing a blueprint for the future I had to disagree with him. "

"Did you inform him of this disagreement?" I ventured to ask.

"Yes, I did, " returned Huxley. "Soon after the publication of *1984* I wrote him a letter in which I praised his novel, but suggested that its horrors were destined to modulate into the comparatively benign brainwashing of *Brave New World.* I observed that the philosophy of the ruling minority in *1984* is a sadism that has been carried to its logical conclusion by going beyond sex and denying it. But I questioned whether in actual fact the policy of the boot-on- the-face could go on indefinitely. I stated my own belief that the ruling oligarchy will find less arduous and wasteful ways of governing and of satisfying its lust for power, and that these ways will resemble those which I described in *Brave New World*. I went on to say that I thought that within a generation the world's rulers would discover infant conditioning and narco-hypnosis to be more efficient, as instruments of government, than clubs and prisons, and the lust for power can be just as completely satisfied by suggesting people into loving their servitude as by flogging and kicking them into obedience. The change will be brought about as a result of a felt need for increased efficiency. Finally I pointed out that there may be a biological and atomic war—in which case we shall have nightmares of other and scarcely imaginable kinds."

"How did Orwell express his disagreement with you?" I asked.

"I knew that Orwell disagreed with me on these questions because a few years earlier he had published a review of the Russian writer Zamyatin's novel *We* which had been written in 1920. In this remarkable novel, which I had not heard of until I read Orwell's review (I read the book itself later), Zamyatin depicts a society within which the idea of scientific management has been carried to its logical end: human beings are not merely subordinated to the machine, but are required to become mechanisms themselves. In his review Orwell compared Zamyatin's book with *Brave New World.* As I recall it, he said that he found my novel less realistic politically than Zamyatin's, since I supplied no clear reason why the society I depicted should be stratified in such an elaborate way. Orwell asserted, correctly, that the aim of the ruling caste in my book is not economic exploitation, and yet the desire to bully and dominate does not seem to be a motive either. In fact, there is no power hunger, no sadism, no hardness of any kind. Those at the top have no strong motive for staying at the top, and though everybody is happy in a vacuous way, life has become so pointless that it is difficult to believe that such a society could endure. Orwell thought Zamyatin's book more relevant to our present situation because of its intuitive grasp of the irrational side of totalitarianism—human sacrifiice, cruelty as an end in itself, the worship of a leader who is credited with divine attributes. It seems pretty clear that Zamyatin's novel had an influence on *1984.* The two show definite similarities. But it did not influence the writing of *Brave New World,* despite Orwell's implicit claim in his review that it had."

"Mr Huxley, that is most illuminating" I said. "But ow, with your indulgence I'd like to turn to the subject of spirituality, which I know you hold to be of supreme importance in our lives. Please correct me if I'm mistaken, but it seems that you didn't always hold this view. Your early novels contain much mockery of religious belief. You are, after all, the grandson of Thomas Henry Huxley, who ridiculed Christian superstitions and suggested that evolutionary science could replace them. Yet in 1945 you published *The Perennial Philosophy,* a work steeped in mysticism in which you attempt to present what you call the Highest Common Factor of all theologies by assembling passages from the writings of those saints and prophets who have approached a direct spiritual knowledge of the Divine. The tone of the work is in striking contrast with the mordant wit and irreverent spirit of your earlier novels. May I ask at what's what point in your life did you begin to move in the direction of spirituality and mysticism?"

"Yes, you're right," returned Huxley, "there is no trace of the spiritual in my first few novels. But in *Point Counter Point* some of the characters do show glimmerings of spirituality. My own first steps on the road to spiritual enlightenment were taken not long after the publication of *Brave New World,* when it dawned on me that in the process of writing the novel I had undergone a kind of subconscious catharsis in which I jettisoned much of my cynical attitude toward spirituality. In fact I began to find myself pulled towards the world-view of John the Savage. You will recall that he was the character I introduced in my novel who, in elevating spiritual values above all else, protests against the soulless pursuit of happiness that he sees as the *raison d'être* of the society of *Brave New World.* I myself came to reject my own youthful view that the world is just a mechanism, a fool of time, devoid of spiritual values and ultimate meaning, and which seemed to be hurtling to destruction. But in my search for deeper values I found myself less attracted to any specific religious doctrine than to the constellation of values common to them all. Leibniz called this the *philosophia perennis* - it is the metaphysic, timeless and universal, that recognizes a divine Reality encompassing the world of things and lives and minds; the psychology that finds in the soul something similar to, or even identical with, divine Reality; the ethic that places man's final end in the knowledge of the immanent and transcendent Ground of all being. "

"Knowledge is acquired in ways which vary with the subject matter", I observed. "In empirical science, knowledge is acquired through experiment; in mathematics, through proof. Self-knowledge is acquired through introspection. How do you think knowledge of the immanent and transcendent Ground of all being is to be acquired?"

"The nature of the transcendent Ground, the one Reality,' Huxley responded, "Is such that it cannot be directly and immediately apprehended except by those who have chosen to fulfil certain conditions, making themselves loving, pure in heart, and 'poor in spirit'. We don't know why this should be the case. It is just one of those facts which we have to accept, whether we like them or not and however implausible and unlikely they may seem. Nothing in our everyday experience gives us any reason for supposing that water is made up of hydrogen and oxygen; and yet when water is subjected to certain efficacious treatments, the nature of its constituent elements becomes manifest. Similarly, nothing in our everyday experience gives us much reason for supposing that the mind of the average sensual man has, as one of its constituents, something resembling, or identical with, the one Reality underlying the manifold world; and yet, when that mind is subjected to certain efficacious treatments, the divine element, of which it is in part at least composed, becomes manifest, not only to the mind itself, but also, by its reflection in external behaviour, to other minds. It is only by making physical experiments that we can discover the intimate nature of matter and its potentialities. And it is only by making psychological and moral experiments that we can discover the intimate nature of mind and its potentialities. In the ordinary circumstances of average sensual life these potentialities of the mind remain latent and unmanifested. In other to realize them, we must fulfil certain conditions and obey certain rules, which experience has shown empirically to be valid. If one cannot fulfill these conditions oneself, the best thing one can do, in the field of metaphysics, is to study the works of those who have achieved this state and who, because they have modified their merely human mode of being are capable of a more than merely human kind and amount of knowledge. It was for this reason that I assembled the quotations that form the greater part of my book."

I said: "I'm particularly interested in how achieving knowledge of the transcendent Ground resolves the traditional mind/body problem that has plagued philosophy - Western philosophy, at least - for so long?" How, that is, is the subjective nature of the individual mind to be reconciled with the objective nature of matter? There have been, in essence, four traditional solutions to this problem. First, the idealist solution: only subjectivity truly exists, the material is no more than a byproduct of mental activity. Second, the materialist solution: only the material world truly exists, mental phenomena are byproducts of physical activity in the brain, thus consciousness, subjectivity and the self are in some way illusions. Third, the dualist solution: there are two substances mind/soul and matter, which exist independently but in some mysterious way interpenetrate in brains. And finally, the Vedantist solution: there is a single undifferentiated reality in which both the subjective and the material are illusions. It seems to me that the Vedantist doctrine is closest to the ideas you have expressed."

"Yes, you are right," Huxley responded. "I do think the transcendent Ground and the Vedantist Brahman - the Ultimate Reality -are one and the same. Of course the traditional schools of Hindu philosophy have differed in their accounts of the nature of Brahman. One school says that Brahman and Atman, the individual consciousness, are identical, another that they are different. But they all share the belief that the Ultimate Reality transcends immediate appearance, that immediate appearance is Maya, illusion."

"Would you allow me, Mr. Huxley," I said, "to present a brief outline of the hybrid form of Vedantism I have formulated in an attempt to resolve the mind/body problem, to my own satisfaction at least? I would be most interested in your opinion."

"By all means, " Huxley responded. "You have my undivided attention."

"You are most accommodating, Mr. Huxley," I said. "Here are the main points of my conception. Objective Reality is fourfold, consisting of space, time, matter, and consciousness. The first three of these components constitute the *Material World.* Each is divisible in the sense of possessing parts – space contains regions, time durations, matter chunks. The fourth component, Universal Consciousness, the Hindu Brahman, is, on the other hand indivisible and possess just a single attribute, namely pure self-awareness. Individual, conscious selves, or minds, are *localizations* (or *manifestations* or *precipitations*) of the Universal Consciousness in the Material World. The indivisible unity of Brahman's self-awareness is inherited by each of its localizations, giving rise to the "unity of consciousness' as noted by Descartes. While the Universal Consciousness itself does not vary, the contents of each localization - each individual consciousness- varies with the individual localization. But localizations of the Universal Consciousness in the Material World occur within *bodies,* more precisely *nervous systems* or *brains.* The more structurally complex the nervous system, the more sophisticated the manifestations of universal consciousness made possible within it. I think a good analogy here is provided by the varieties of pattern produced by water passing through fountains. The water itself does not vary in its essence, but the patterns induced in the water by such passage can be infinitely varied and complex. The extraordinary biochemical complexity of the brains of organisms, in particular the human brain, is *necessary* - no simpler physical structure is capable of producing the complexity of the process of thinking, the manifestation of the Universal Consciousness within the brain. The subjectivity of individual minds - the awareness of selfhood - arises as the localization of the universal *I am,* the self-awareness of the Universal Consciousness. Just as fluidity is the sole attribute the water contributes to a fountain's display, self-awareness is the sole attribute the Universal Consciousness bestows on individual minds. The fact explains why individual minds cannot communicate directly with one another and why babies only come to grasp the idea of other minds as they grow up. Apart from self-awareness, all attributes of individuals arise through the spatio-temporal -material particulars of that individual. The Universal Consciousness can thus be thought of as a very simple genotype and the contents of individual consciousnesses, or individual minds, as varying phenotypes."

"The Universal Consciousness," I went on, "having only the attribute of self-awareness, is entirely beyond the Material World of space, time, and matter. But its localizations as individuals in the Material World are aware of location in space relative to other individuals as well as the passage of time. The latter is experienced as a continuous flow through what we call the 'Present'. Only individual consciousnesses have such experience; the Universal Consciousness has no 'experience' except self-awareness. The Universal Consciousnesses is beyond the Material World. Only its individual manifestations in the Material World are subject to the constraints of, and the latent possibilities within, the realm of space, time and matter."

"Those are the main points of my conception of realit,y,"I concluded.

At this point I ceased talking, and waited for Huxley to respond. After a few moments he got up from his chair and paced about the room in thought. Then he returned to his chair, looked me in the eye, and said, "Your disquisition was most interesting, worthy of a professor of philosophy. I shall have to renew my subscription to *Vogue!* Please correct me if I am wrong, but I suspect you arrived at your conclusions by rational analysis rather than by revelation. I don't mean this as a criticism of your analysis, on the contrary I think it's very insightful, but, unlike truth grasped by revelatory means, the truth of conclusions arrived at by rational analysis depends on the truth of the premises from which one starts. And this truth requires *evidence.* The chief premises on which your analysis is based are the existence of space, time, matter and the Universal Consciousnesses. The existence of the first three is based on extrapolation from the immediate evidence of the senses. Am I correct in supposing that the evidence you would adduce for the existence of the Universal Consciousness is a similar, subjective extrapolation from the immediate access one has to one's own awareness?"

"Yes, you are quite right," I responded. "I compare it to the extrapolation science makes from empirical facts to the existence of subatomic particles, except that the subjective extrapolation from immediate evidence of awareness to the existence of the Universal Consciousness is infinitely simpler. It is logically possible for only the Material World to exist, in which case the Universe would be nothing but a blind mechanism. But if this were truly the case, it must follow that subjective awareness, whose existence is immediately evident to an individual mind is constructible from the constituents of the Material World. I find it impossible to see how the subjectivity of conscious awareness can be constructed from purely material elements. Materialists who accept this impossibility but who refuse to accept the independent existence of the mind merely declare awareness and consciousness to be illusions generated by brain activity. You might as well declare gravitation to be an "illusion"! No, the one thing of whose existence I am absolutely certain is my own consciousness. Sartre has put this conviction decisively: *Before there can be any truth whatever, then, there must be an absolute truth, and there is such a truth which is simple, easily attained and within the reach of everybody; it consists in one's immediate sense of one's self."*

"Sartre's declaration is indeed very wise, " Huxley rejoined. " But I don't think that one's immediate sense of oneself, of one's own consciousness, furnishes sufficient evidence for the existence of a Universal Consciousness. As you have admitted, you have arrived at that conclusion indirectly, by means of rational analysis, starting with the incontrovertible evidence of the existence of one's own consciousness. I have long sought direct evidence of the presence of the Universal Consciousness, an evidence as immediate as the incontrovertible evidence of the existence of one's own consciousness given through introspection. The closest I have come to achieving this blissful state is through the ingestion of the psychedelic drug mescalin. As I reported in my book *The Doors of Perception,* the great change induced by taking the drug was not in my inner state but rather in the realm of objective fact, in what I actually saw. The other world to which mescalin admitted me was not the world of visions; it existed out there, in what I could see with my eyes open. Objects glowed with an inner light which I saw as a direct manifestation of the Universal Consciousness. Yet at the same time I was fully aware that my admission to this extraordinary transcendent world was the result of the change in my brain chemistry caused by the psilocybin. This only confirmed to me that one can only become directly aware of the presence of the Universal Consciousness when one's brain is in a particularly receptive physical state. This seemed, and still seems, perfectly reasonable to me."

"Yes, I agree, direct evidence of existence is the gold standard, " I said. "I envy you that you have come close to finding direct evidence for Universal Consciousness. Sadly, I expect I shall have to remain content with the indirect variety. "

At this point I got up from my chair and drew the interview to a close. "Mr. Huxley, I have greatly enjoyed our conversation, which, thanks to your graciousness and insight, has fulfilled my every expectation. I thank you most sincerely."

Huxley also rose, and as we shook hands he said, with a smile, "I too have enjoyed our time together. May I leave you with the friendly advice to take a dose of mescalin , under controlled conditions, of course, if you cease to be contented with indirect evidence of the Universal Consciousness."

"Mr. Huxley, I shall take your advice most seriously. Farewell."

After leaving Huxley's apartment, I returned to my hotel room and retrieved the copy of Luria's book which I dad left there. On the flyleaf I inscribed "For Aldous Huxley from John Bellane, in appreciation of a memorable conversation." I wrapped it in brown paper and addressed the parcel to Huxley at his apartment. I left the hotel and proceeded to the nearest post office, where I mailed my parcel. I then returned to the hotel and paid my bill. Finally I returned to my room, packed my bag, and time-travelled back to 1910 London. My encounter with Huxley had indeed been memorable.

Fred Hoyle

In the course of my researches into the cultural developments of the 20th century I had been particularly struck by the remarkable efflorescence of scientific cosmology. What had been since Newton’s time an esoteric pursuit on the fringes of astronomy, the 20th century saw cosmology blossom into a grand intellectual game, an intoxicating mixture of physics, mathematics and philosophy, flavoured with liberal doses of slheer speculation, that came to grip the public imagination.

This development, I learned, had been sparked in 1917 by Einstein’s application of his recently formulated general theory of relativity to elucidate the structure of the physical universe as a whole. Two centuries earlier, when Newton applied his theory of gravitation to explain the structure of the universe, he had made what seemed the natural assumption that the universe was static in the sense that its large-scale features did not change over cosmic time. Einstein retained Newton’s assumption, in the first place because, apart from local motions, the universe of “fixed” stars appeared to be static; and also because, as Einstein remarked, “one would get into bottomless speculations if one departed from [that assumption].” But the universe that resulted from Einstein’s investigations was merely static, it was also finite. Like Newton before him, Einstein was then faced with the difficulty that mutual gravitational attraction would cause stars to clump together over time. Even more drastic was that according to Einstein’s theory gravity would cause a universe containing matter to contract as a whole and ultimately collapse altogether. To counteract this he introduced a force of “cosmic repulsion” into his mathematical model. The original universe envisaged by Einstein is static, finite and like the surface of a sphere, unbounded.

In the 1920s the static view of the universe was overturned by two discoveries, one theoretical, the other empirical. In 1922 the Russian mathematician Alexander Friedmann, and, independently, the Belgian cleric Georges Lemaître in 1927, showed that Einstein’s equations have solutions which represent a universe containing matter, but in a state of continual expansion. In 1929 Edwin Hubble, in apparent ignorance of Friedmann' and Lemaître’s theoretical insights, found the first evidence of such an expansion, observing a red shift in the spectrum of galaxies in direct proportion to their distance from us. Hubble’s observations seemed at first to indicate that the expansion of the universe had begun only a billion or so years ago, contradicting the evidence from radioactivity in rocks that the earth’s crust must be at least 5 billion years old (later revised to 4.5 billion years). Happily, this conflict was resolved by a later revision of the distance yardstick based on stellar luminosity, which resulted eventually in the origin of the expansion being pushed back to a point some 14 billion years in the past.

The discovery that the universe was expanding naturally led to the question of when and how the expansion began. Lemaître suggested that the universe had evolved by explosive expansion from an initial highly compressed and extremely hot state which he called the "primeval atom"—the first explicit formulation of what later came to be known as the "Big Bang" scenario, in which the universe has a "beginning" of some kind. The apparent support of science for this idea naturally appealed to cosmologists of a Christian persuasion, who held that God had created the universe *ex nihilo.* Understandably, cosmologists and physicists of an atheistic tendency became alarmed that their discoveries could be used, plausibly or not, to support traditional Christian theology. Unwilling to become embroiled in theological dispute, many physicists welcomed the formulation of the steady-state theory of the universe put forward in 1948 by the physicists Hermann Bondi, Thomas Gold and Fred Hoyle. This provided an alternative to the "Big Bang" scenario—a term introduced with derisory intent by Hoyle in 1950, which ironically caught on—according to which the universe apparently sprang explosively into being from nothing and then expanded, continually cooling and attenuating, into its present quiescent state. In the steady-state theory, by contrast, it is denied that the universe is any cooler or less dense at present than it was in the past. It is further denied that any change in the large-scale structure of the universe has occurred over time. Most importantly from a philosophical standpoint it is denied that there was ever a time in the past at which the universe has not existed. These assertions were based on what Bondi and Gold termed the Perfect Cosmological Principle. This is an extension of the fundamental Cosmological Principle - accepted by virtually all cosmologists - which asserts that, on a sufficiently large scale, the properties of the universe are the same for all observers, wherever they happen to be situated, although these features my change over time. The absence of such an assumption would make observational cosmology virtually impossible, for without it very little could be inferred about the structure of the universe as a whole on the basis of what can be observed from the tiny part of the universe occupied by the Earth. The Cosmological Principle allows for large-scale changes in the structure of the universe over time. Bondi and Gold's Perfect Cosmological Principle extended the Cosmological Principle to time as well, so asserting that, on a sufficiently large scale, the properties of the universe are the same for all observers, wherever and whenever they happen to be situated. According to the Perfect Cosmological Principle the universe looks the same everywhere (on a sufficiently large scale), and its appearance does not change with time.

At first sight, the Perfect Cosmological Principle conflicts with the observed expansion of the universe. The devisers of the steady-state theory were well aware of this. They saw that the only way of preserving a fundamentally changeless universe undergoing expansion was to postulate what Hoyle termed the continuous creation of matter, that is, a continual uniform creation of matter at precisely the rate required to offset the attenuation brought about by the expansion. In order to compensate for the universe's expansion the steady-state theory called for the "creation" of just one hydrogen atom per cubic centimerer of space every quadrillion years. This is a phenomenon well below the limits of conceivable observation. Cosmologists anxious to steer clear of the theological quagmire caused by the Big Bang theory were more than willing to accept a minuscule amount of "continuous creation" of matter in order to avoid the problem of the universe's origin.

While the steady-state theory did not lack philosophical appeal, the observational evidence soon began to mount against it. By the mid-1960s most cosmologists regarded the steady-state theory as dead and buried. Nevertheless, despite the mounting observational evidence in favour of the Big Bang theory, Hoyle himself steadfastly refused to accept it. Hoyle's stubbornness in this regard was satirized in an amusing verse by Barbara Gamow, the wife of the physicist George Gamow, himself a champion of the Big Bang theory. Here Ryle is the British radio astronomer Martin Ryle, who, as a proponent of the Big Bang theory, engaged in extended debate, much of it acrimonious, with Hoyle throughout the 50s and 60s.

"Your years of toil,"
Said Ryle to Hoyle,
'Are wasted years, believe me.
The steady state
Is out of date.
Unless my eyes deceive me,
My telescope
Has dashed your hope;
Your tenets are refuted.
Let me be terse:
Our universe
Grows daily more diluted!"
Said Hoyle, "You quote
Lemaitre, I note,

And Gamow. Well, forget them!
That errant gang
And their Big Bang –
Why aid them, and abet them?
You see, my friend, it has no end.
And there was no beginning,
As Bondi, Gold,
And I will hold
Until our hair is thinning!"

Hoyle's unorthodoxy led to his being described as "the greatest scientific rebel of the late 20th century." In the early part of his career he made a number of impressive scientific discoveries, the most important of which was his contribution to solving the problem of stellar nucleosynthesis, that is, the problem of how stars create heavier elements, such as carbon, nitrogen and oxygen. Stars derive their power from the fusion of basic elements, and in the process new elements are generated. But it was only when Hoyle identified a key link in the process - a special state of carbon-12 - that the production of the full sequence of elements could be understood. One of Hoyle's collaborators received a Nobel Prize for their work, but, controversially, Hoyle himself was passed over by the Nobel committee.

As he grew older Hoyle came to espouse theories of increasing eccentricity. The most controversial of these was his contention that organic life did not originate on earth, but arose rather as the result of bombardment of the earth's surface by micro-organisms or biochemical compounds from outer space. These micro-organisms, according to Hoyle, permeate the universe and constitute what he called "Panspermia". Hoyle also questioned Darwin's theory of natural selection, asserting that, on statistical grounds the unlikelihood of complex life evolving in the way it has through natural selection alone was so great that it should be rejected. Hoyle believed that some additional mechanism must be involved in the evolution of life, and he was unabashed to speculate that this mechanism could be the product of intervention by a higher intelligence.

I was eager to engage this controversial genius in conversation. In 1988, at the age of 73, he and his wife Barbara had settled in Bournemouth on the southern coast of England. I thought that this would offer a suitable venue for our meeting.

I decided to present myself as a staff member of *Omni* magazine. Accordingly In July 1991 I time-travelled to Bournemouth and took a room in a hotel overlooking the seashore. I found Hoyle's telephone number in the directory and dialled it on the room phone. After a few rings a man came on the line and said, in an unmistakable Yorkshire accent, "Who's this, then?" "Am I speaking to Professor Hoyle?", I responded. "Yes, you are. Ex-professor, any road. And who am I speaking to?" "My name is John Bellane, and I'm a staff member of *Omni* magazine. I'd very much like to arrange an interview with you." "*Omni*, eh? Isn't that the magazine mixing up popular science, science fiction and allsorts?" "Well, yes, that's about right." "In that case, I'm happy to talk to you. I've published SF myself, you know." "I'm very well aware of that, sir. When would it be convenient to meet?" "Could you come to my flat the day after tomorrow at 2 p.m.?' "I'll be there."

At the agreed time I knocked on the door of Hoyle's apartment, which proved to be located on the top floor of a high-rise. The door opened and I was invited in by Barbara Hoyle. As I entered I slid my hand into the pocket of my jacket and switched on my recorder. Mrs. Hoyle conducted me into a spacious living room from which the sea coast could be seen through the wide windows. Hoyle sat in an armchair intently watching a cricket match on television. His eyes still glued to the screen, he rose and shook my hand abstractedly. His wife, gently pointing out to him that he had a guest, went over to the television set and turned it off. At this point he turned his full attention to me.

He gestured me to an armchair next to his. I said to him, "Professor Hoyle, thank you for agreeing to this interview."

He nodded and replied, in the Yorkshire accent he had never lost, "Well, I hope it proves productive, any road."

I decided to get straight to the point. " In the 1950s and 60s your books and broadcasts on cosmology and astrophysics made you a virtually a household name. It is almost as if the well-worn phrase 'according to Hoyle' had acquired a new meaning! You brought the esoteric subject of cosmology to the general public through your spirited defence of the idea, originated by you and your colleagues Hermann Bondi and Thomas Gold, that the expanding universe has no origin in time - that it has always existed in the past in more or less the same state it is now and will continue to be in the future. Opposed to this is the conception, proposed by cosmologists such as Lemaitre and Gamow, that the universe suddenly sprang into being as an exploding fireball at some time in the past. The first of these alternative accounts of the universe - what might be called the Parmenidean picture - came to be known as the 'steady-state' theory and the second- the

Heraclitean picture - you memorably termed, in an early radio broadcast, the 'Big Bang' theory. Could you explain what it was that drew you to the first of these conceptions rather than the second?"

Hoyle looked at me quizzically and responded: "Ah, I see that you're attracted to philosophy. Parmenides, as I recall, was the Ancient Greek philosopher who claimed that nothing changed and Heraclitus was a contemporary who claimed that everything was born in fire and was constantly changing, that you can't step twice into the same river. Well, I have to say that, while I can hardly claim to be a philosopher, in my youth I had come to reject the biblical claim that God created the universe from nothing, since this raised the obvious question of the origin of God. It seemed natural to me to avoid the problem of creation, and the existence of God in the traditional sense, by denying that the universe had to be 'created' at all. In other words, the universe has always existed - it has no beginning, there was no moment in the past at which it was brought into being. Unlike Parmenides (of whom I hadn't then heard), I didn't see the universe itself as unchanging. But once I had read a little philosophy I was attracted to Parmenides' view since it accorded with my basic conviction that the universe has no beginning. As a physicist, I was repelled by the 'Big Bang' conception of the universe as being created without a pre-existing space and time in which that creation could take place. It would be as if a painter could create a picture without a canvas! It seemed to me that to accept this idea would thrust the "creation" beyond physics altogether, into the realm of theology. Certainly the universality of the laws of physics would be lost. "

"Your conviction is that space and time must always have existed," I said. ''What about matter? I suppose that you do not accord it the same eternal status as space and time since in your version of the steady-state theory you postulate the 'continual creation' of matter in order to compensate for its attenuation as the universe expands. "

Hoyle nodded. "But of course ever since Einstein invented the special theory of relativity with its equivalence of mass and energy, matter has ceased to occupy the central position it held in classical physics. And in modern cosmology, whose mathematical formulation is based on the general theory of relativity, the form of the universe is the geometry of space-time as determined by the gravitational field, rather than directly by matter itself. Thus general relativity allows for universes which contain no matter whatsoever, but in which space and time are ever present. So while I reject the idea of space and time being 'created', I could see no objection in principle to the idea of new matter (or its equivalent, energy) coming into existence within a pre-existing space and time. In my original version of the steady-state theory I introduced a universal scalar field - the C or 'creation' field - into relativistic cosmology to represent the continual creation of matter. The C-field amounted to a reservoir from which matter can be created. It has the simplest physical laws imaginable - even simpler than those of the electromagnetic field. Somewhat like the lilies of the field of Biblical fame, it has no mass, no charge and no spin. It comes into effect only at the time when particles are created. When a particle with a certain energy is created, a C-field of equal but negative energy is also radiated. The overall energy is therefore conserved. This solution had, I thought, the advantage over the big-bang models in that there is no singular point where the mathematical and physical descriptions break down. The steady-state model asserts that although the universe is expanding, it nevertheless does not change its appearance over time: has no beginning and no end. And as you have pointed out, this required that matter be continually created in order to keep the universe's density from decreasing. "

"I'm sure you'll recall your 1950 radio broadcast in which you offered a spirited defence of the hypothesis of continual creation. Here is what you said." I pulled out my notebook and began to read from it. *Some people have argued that continuous creation introduces a new assumption into science – and a very startling assumption at that. I do not agree that continuous creation is an additional assumption. It is certainly a new hypothesis, but it only replaces a hypothesis that lies concealed in the older theories, which assume ... that the whole of the matter in the universe was created in one big bang at a particular time in the remote past. On scientific grounds this big bang assumption is much the less palatable of the two. For it is an irrational process that cannot be described in scientific terms. Continuous creation, on the other hand, can be represented by mathematical equations whose consequences can be worked out and compared with observation. On philosophical grounds, too, I cannot see any good reason for preferring the big bang idea. Indeed, it seems to me in the philosophical sense to be a distinctly unsatisfactory notion, since it puts the basic assumption out of sight where it can be challenged by a direct appeal to observation."*

"Yes, I do recall that observation" Hoyle affirmed. "My basic objection to the Big Bang model has always been that its postulation of a 'creation' from nothing of the universe, of the whole kit and caboodle of space, time and matter places that creation not just beyond physical observation, but also beyond mathematical analysis. Now it is true that the continual creation process of steady-state theory, which demanded the 'creation' of just one hydrogen atom per cubic centimetre of space every quadrillion years, is itself a phenomenon well below the limits of conceivable observation. But that process has a perfectly satisfactory mathematical formulation in the concept of a C-field, whose consequences can be worked out and compared with observation. This is in principle no different from the kinetic theory of gases in the nineteenth century which was a mathematical theory explaining the behaviour of gases based on the motions of unobservable molecules.

“We musn’t forget that the Big Bang theory had also acquired theological baggage. Pope Pius XII wholeheartedly approved of it, declaring it to offer scientific proof for the existence of God. But those cosmologists wanting to steer clear of the theological quagmire caused by the Big Bang theory were more than willing to accept an miuiscule amount of ‘continuous creation’ as the price to be paid for thrusting the origin of the universe back to minus infinity, where they instinctively felt it belonged. I was pleased when Steven Weinberg, despite his support for the Big-Bang theory, declared that the steady state theory is philosophically the most attractive theory because it least resembles the account given in Genesis.”

“Yes , the steady-state theory was, and is, philosophically appealing, “ I said. “But you must admit that observational evidence has mounted against it. Allow me to recapitulate some of that evidence. To begin with, continuous creation required particles and corresponding anti- particles, such as electrons and positrons, to be produced at equal rates, which would lead to a symmetry between matter and antimatter. But the observed universe shows no such symmetry. Rather it shows a marked preponderance of one sort of matter over the other. Moreover, the discovery of quasi-stellar objects (‘quasars’) at great distances showed that the universe did, after all, change its appearance with time. The Big Bang theory received further confirmation over its rival from the successful prediction of the cosmic abundances of the light elements helium, deuterium, and lithium, all of which would be produced by nuclear reactions during the first three minutes of the expansion after the Bang. The steady-state theory was incapable of explaining the abundance of these elements. But by far the most telling piece of evidence in favour of the Big Bang theory was the discovery of what became known as the ‘echo’ of the Big Bang. Gamow and his collaborators had predicted that if the Big Bang scenario of a hot and dense past were correct, then some evidence of that past must remain in the form of residual radiation cooled by the universe’s expansion to a temperature only a few degrees above absolute zero. In 1965 Penzias and Wilson happened upon this radiation field while calibrating a sophisticated radio receiver designed for satellite tracking. The radiation had a temperature of three degrees absolute, almost exactly as predicted. Subsequent observations established that its spectrum carried the distinctive Planck signature of heat radiation. The steady- state theory provided no plausible means of explaining the presence of a pervasive radiation field with just these characteristics.

“Would you care to comment on all this?” I finished up.

Hoyle nodded and said, “Well, you have certainly done your homework! Yes, the pieces of observational evidence you have mentioned were important and they did fit nicely into the Big Bang picture, especially the abundance of light elements and the 3 degrees K. background radiation. My collaborators and I acknowledged this and responded by developing a modified version of steady-state cosmology which we called quasi-steady-state cosmology (QSS). This model retains the eternality of the universe central to the steady-state picture but incorporates ‘pockets’ or ‘bubbles’ of creation appearing over time, which we termed *mini-creation events,* or *little bangs.* In this modified picture the structure we ordinarily think of as ‘the Universe’ is but part of a larger whole. It is then natural to ask – if there is a larger Universe ‘outside’, can we learn anything of its structure? What kind of information could we use to come to grips with such a question? ... The physical laws themselves – as we determine them experimentally – may have much to tell us about the nature of the larger Universe.”

At this point I ventured to observe: “So this larger universe can be thought of, in effect, as an eternally existing overarching ‘multiverse’ composed of numerous ‘bubbles’ each arising through a ‘minibang’ and each developing into a universe of its own, a ‘local’ universe, as it were. And wouldn’t the universe we inhabit be one of these?”

“Yes, that’s right,” Hoyle responded.

“In that case,” I continued, “Wouldn’t it be natural to explain the nearly Planckian spectrum of the cosmic microwave background observed in ‘our’ universe as the result of the explosive minibang which initiated it? This explanation seems preferable to the somewhat ad hoc ‘dust-grain’ hypothesis put forward by you and your collaborators to account for the cosmic microwave background.”

‘’We have entertained that idea,” Hoyle said, “and I agree that it is attractive. QSS is, after all, an attempt to modify the stray-state theory to accommodate the new pieces of observational evidence that were believed to be better explained by Big Bang than by steady- state. The discovery of the Planck spectrum and homogeneity of the cosmic microwave background was the major reason why a number of cosmologists ceased to be steady-statesmen and joined the Big Bang camp. But the Big Bang picture also had, and has, certain observational deficiencies in addition to its postulation of a nonphysical ‘beginning’ of the universe. These, as you probably know, arise from the so-called ‘horizon’ and ‘flatness’ problems. The first of these is the puzzle that widely separated regions of the universe are observed to share the same physical properties, such as temperature, even though these regions were too far apart when they emitted their radiation to have exchanged heat and so homogenized during the time since the Big Bang. The ‘flatness’ problem is the question of why the universe today is so close to the boundary between being open or closed, that is, why it is almost ‘flat’. While these features arise naturally in the steady-state picture, in which past time is infinite, they present genuine difficulties for any cosmogony postulating a finite past time. As a matter of fact, Newton encountered a similar difficulty with his own cosmogony. Newton came to realize that, if the universe contained only finitely many stars, then, given sufficient time, universal gravitation would cause the stars to clump together under their mutual attraction. Since this has no happened, it follows either that there has not yet been sufficient time since the creation of the universe for the clumping to occur, in which case the universe has o finite past time. The other possibility is that there the universe contains an infinite number of stars homogeneously distributed so as to be in gravitational equilibrium, thereby avoiding being clumped together. Newton wisely opted for the latter possibility.”

“Thus Newton chose what might be called the ‘escape into infinity’,” I interjected. “And the Big Bang advocates…?”

“Well, the Big Bang advocates could not abandon their obsession with a finite past time,” Hoyle continued, “and so had to come up with ingenious but , in my view, unacceptably *ad hoc* explanations for horizon and flatness. The most popular of these is the inflationary universe model devised by Alan Guth and others. The essential feature of the inflationary universe model is that, shortly after the Big Bang, the infant universe underwent a brief (perhaps as little as 10–32 sec.) and extremely rapid expansion, after which it resumed the more leisurely rate of expansion of the standard Big Bang model. This hypothesis does indeed ‘resolve’ the horizon and flatness problems but seems completely ad hoc, comparable to the epicycles of Ptolemaic cosmology. What inflation actually amounts to is the postulation that, soon after the Big Bang, the speed of light underwent a sudden increase for a very brief time, and then fell back to its original value. This is, of course, logically possible, but, apart from providing an ‘explanation’ for horizon and flatness, it is not justified on scientific grounds. One might as well say that God, having created the universe with the Big Bang, saw that, without a temporary increase in the speed of light, his creation would be chaotic, lacking the harmony which had inspired his original vision. All very dramatic, but hardly scientific.”

“Your principal objection to the Big Bang picture,” I observed, “has always been that it postulates a ‘beginning’ to the universe, before which the universe itself did not exist. This ‘beginning’ has been identified as a singularity, a place where the laws of physics break down in some sense. Now Hawking and Penrose have shown that, under certain seemingly plausible hypotheses within the general theory of relativity, there a ‘universal’ singularity must be present, a single point in the past of the universe at which all past-directed spacetime paths terminate. Hawking traced this back in time to the Big Bang, which he claims is a point of infinite density, marking the ‘beginning’ of the universe in time. What do you make of this?”

After pondering for a moment Hoyle replied: “I think declaring that the laws of physics break down at spacetime singularities is to overdramatize the situation. In mathematical analysis a singularity of a curve, say, is a point where the curve has a discontinuity or a cusp, that is, a point where the curve fails to have a certain property, such as continuity or differentiability, which it possesses almost everywhere else. In such cases we don’t say that the laws of mathematics have ‘broken down’, but rather that the curve in question is of a singular type. Analogously, the ‘laws of physics’ are our guesses that hold good in certain regimes, say of temperature or density. When these limits are crossed, the ‘laws of physics’ naturally give nonsensical answers and need to be replaced with a better guess, which we call new ‘laws’. In physics this is what singularity means--an extrapolation made using a guess that gives a nonsensical answer. All it means is that a better guess needs to be made. A mathematical singularity has no connection with the ‘creation of the universe’. This point cannot be emphasized strongly enough. Physics is not competent to pronounce on the creation of the universe. Physics requires a running universe even to start. All physics does is to correlate a configuration of particles or fields at one time with another configuration at a different time. This makes no sense if there are no particles or fields to begin with. Hawking himself now claims merely that general relativity breaks down at times prior to the Big Bang, and hence no singularity could be predicted by it. “

“It seems to me,” I said, “that in rejecting the Big Bang, and cleaving to your conviction that the universe has no beginning you have been able to avoid the more fanciful conjectures of the Big Bang advocates who, while accepting the finitude of the past, are reluctant to surrender the problem of the origin of the universe to the theologians and philosophers. Instead they want to treat the spontaneous emergence of the universe as a problem in physics. They hope that a suitable synthesis of quantum theory and relativity would enable the derivation of the initial singularity to be blocked. In that case, the putative ‘beginning’ or ‘creation ex nihilo’ of the universe need no longer be identified with something as nebulous as a singularity, but could actually be invested with physical content. What do you think of this idea?”

“Well,“ Hoyle replied, “I must acknowledge the ingenuity of some of these Big Bangsters. My personal favourite is the proposal put forward, almost certainly with tongue in cheek, by Edward Tryon. Breathtaking in its simplicity, his suggestion was that the universe may be nothing more than a monstrous ‘vacuum fluctuation’ in the sense of quantum field theory. A typical example of a vacuum fluctuation is the occasional emergence of an electron, positron, and photon from a perfect vacuum. When this occurs, the three particles exist only for a brief time, and then annihilate each other, leaving no trace behind. Energy conservation is violated, but only for the brief particle lifetime permitted by the ‘time/energy’ formulation of the Heisenberg uncertainty principle. This has the consequence that the smaller the net energy of the particles, the longer the fluctuation can last. In particular, if the net energy is zero, then the particle lifetime can be any value whatsoever, however large. Now the laws of physics place no limit on the scale of vacuum fluctuations. So if the universe is closed and has zero net energy (for this to be possible the universe’s total ‘positive’ mass energy must be balanced by its total ‘negative’ gravitational potential energy) it could be itself the result of a vast fluctuation of the vacuum of some hyperspace—the ‘quantum void’—in which our own universe is embedded. I was tickled by his explanation for this remarkable occurrence, that our Universe is simply one of those things that happen from time to time. Amusing, but absurd, of course.

“Hartle and Hawking put forward a more sophisticated, but in my view equally implausible proposal for avoiding the initial singularity. They called their proposal the ‘no boundary’ scenario. They claimed that the universe’s initial state is timeless, in that it possesses, not three spatial and one temporal dimension, but four spatial dimensions, the additional spatial dimension being called by Hawking ‘imaginary time.. Of course, the idea of ‘imaginary time’ was first proposed by Minkowski in 1908 in order to allow the metric of special relativity to assume a Euclidean form. Hawking wrote a book promoting his theory, but despite its being a bestseller, I doubt that its readers really understood what he was getting at. By comparison with steady-state or straightforward Big Bang cosmology his ‘no-boundary’ scenario is highly esoteric. Hawking proposes that in his ‘imaginary time’ the universe has no beginning and yet in ‘real’ time it does. He says that imaginary time is at ‘right angles’ to real time. The purpose of all this ingenious mambo-jumbo is, apparently, to enable to avoid the laws of physics breaking down at the initial singularity whose physical existence the Big Bang makes unavoidable. When I read Hawking's book I was reminded of Lev Landau's observation that cosmologists were often mistaken but never in doubt!"

"With your indulgence," I said, "I'd like now to discuss your contention that life on earth did not originate here but arose as micro-organisms or biochemical compounds from outer space. And you further maintain that, once life was established here, it did not evolve according to Darwin's theory of natural selection, but is driven by the continued impact of such extraterrestrial micro-organisms. What led you to these highly unorthodox conclusions?"

"Well," Hoyle responded, "As common sense would suggest, the Darwinian theory is correct in the small. Rabbits come from other slightly different rabbits, not from either soup or potatoes. Where they came from in the first place is a problem yet to be solved, like much else of a cosmic scale. I have long disbelieved in the notion that that life on earth, or anywhere else, arose out of an abiological organic soup through the development of a primitive replicating system. To begin with, the likelihood of the formation of life from inanimate matter is of the order of one over a number with 40,000 noughts after it, that is, vanishingly small. Also, a primitive replicating system could not have copied itself with anything like the fidelity of present-day systems. With only poor copying fidelity, a primitive system could carry little genetic information without becoming unbearably large, and how such a primitive system could then improve its fidelity and also evolve into a sexual system with crossover beggars the imagination. I am convinced that the Darwinian theory is wrong, and that continued adherence to it is an impediment to discovering the correct evolutionary theory. In fact I would go so far as to say that the believers in neo-Darwinism are suffering from a form of mental illness. Because the old believers said that God came out of the sky, thereby connecting the Earth with events outside it, the new believers were obliged to say the opposite and to do so, as always, with intense conviction. Although the new believers had not a particle of evidence to support their statements on the matter, they asserted that the rabbit producing sludge (called soup to make it sound more palatable) was terrestrially located and that all chemical and biochemical transmogrifications of the sludge were terrestrially inspired. Because there was not a shred of evidence to support this view, new believers had to swallow it as an article of faith, otherwise they could not pass their examinations or secure a job or avoid the ridicule of their colleagues. So it came about from 1860 onward that new believers became in a sense mentally ill, or, more precisely, either you became mentally ill or you quitted the subject of biology, as I had done in my early teens. The trouble for young biologists was that, with everyone around them ill, it became impossible for them to think they were well unless they were ill, which is a situation you can read all about in the columns of *Nature*."

"And how," I asked Hoyle, "did you confirm what you saw as the underlying flaws of Neo-Darwinism? By means of a mathematical analysis, I don't doubt."

"Yes, of course," Hoyle rejoined. "It's the only secure way. I went back to the mathematics behind the Neo-Darwinian theory, of Fisher, Haldane, Sewall Wright and Kimura. Compared with general relativity, the maths wasn't difficult. But my experience proved unrewarding. While the mathematics was comparatively simple, I found the words in which the mathematics was explained to be quite baffling. At first I thought that fault was mine, but as the frustrating sessions with the authors' books were repeated again and again over a period of years, I came to suspect that the confusion was in the heads of the writers themselves. Eventually therefore, I decided to tackle the mathematics myself working from scratch. Although my results were arrived at independently, some-perhaps most have been obtained before. Their arrangement, however, is I believe original. I found that Neo-Darwinism worked only for small-scale changes, and even then only weakly in the general situation. My impression is that some evolutionists have sought to speed things up by wrongly considering cases where species are only coping with environmental conditions they have experienced before, so that memory is being misinterpreted as discovery. The peppered moth, *Biston betularia,* so called because it has speckled black and white wings, is frequently misinterpreted in this sense.

"I also found that it would not even work at all for organisms above the level of bacteria if it were not for an exquisite system of sexual recombination and crossover, used by organisms higher than single-celled organisms. To have any hope of success the neo-Darwinian theory must therefore appeal to a reproductive model quite different from the model mostly adopted by single-celled organisms. This is already an immense climb down from what is usually claimed for the theory. Gone is its 'obvious' status. Only if a model can be found that contrives to uncouple the selective properties of one gene from another, permitting the occasional good mutation to survive and prosper in a sea of bad mutations, can evolution be made to work at all. Such a model can be found. It is exquisitely complex at a biochemical level and yet compellingly simple at a biological level."

At this point Hoyle paused, which I took as an invitation to pose the natural question: "And what would that model be?"

"Sexual reproduction and the resulting chromosomal crossover," Hoyle replied. "But this shows that the supposed logical inevitability of the theory of evolution by natural selection is quite incorrect. Just the reverse is the case. It is only when the present asexual model is changed to the sophisticated model of sexual reproduction accompanied by crossover that the theory can be made to work. But this raises what I consider to be the insuperable problem, namely, how could this exquisitely complex system have arisen from an organic soup in the first place? In effect, we are confronted with another level of irreducible complexity. If single-celled organisms don't use sexual reproduction with crossover (and are reproductively highly successful without it), and everything above single-celled organisms does use it, then how and why did single-celled organisms ever invent such an exquisitely complex system, and why don't we find them still using it today?

"I think that the Neo-Darwinists have concealed how crucial sexual reproduction with crossover is to their entire system. This latter advantage of sexual reproduction seems to be the strongest argument claimed for it over the asexual model. Fisher's *The Genetical Theory of Natural Selection* carries the point in the exquisite ellipticities that were so characteristic of Fisher. With quite some searching one can find it in Sewell Wright's treatise in four volumes *Evolution and the Genetics of Populations* and more directly and clearly in J. Maynard Smith's *The Evolution of Sex*. What one does not find, however, is an appreciation of the really crucial aspect of the matter, that only with sexual reproduction accompanied by crossover can positive mutations make headway against the deleterious mutations which occur with far greater frequency, and which otherwise would swamp the advantageous mutations, not permitting them to make any headway at all."

"You claim that neo-Darwinism only works for small-scale evolutionary changes ," I observed. "In that case, to what do you attribute large-scale changes? What is the source of the new genetic material required for such changes?"

"There seems to be no alternative but to attribute such changes to extraterrestrial sources," Hoyle responded. "Perhaps regular 'genetic storms' from outer space add whole batteries of new genes. I admit the situation isn't clear. But I'm convinced that the new genetic material required for large-scale evolutionary change to occur could not have arisen on the earth. There just hasn't been enough time."

“But if the genetic components of this ‘exquisitely complex’ system could not possibly arise on Earth, how could they arise in outer space? “

“Well,” Hoyle responded, “One possibility is that life already exists in the universe on a grand scale. In the steady-state universe with its infinite past life could have emerged on a random basis - there was enough time for this highly improbable event to occur. The comparatively short past of the Big Bang universe makes the emergence of life on a random basis most unlikely.”

“Be that as it may,“ I said, “ It would seem that you do not think that the beginning of life was a random occurrence. In which case,….”

“That’s right,“ Hoyle interjected, “Once we see that the probability of life originating at random is so utterly minuscule as to make the random concept absurd, it becomes sensible to think that the favourable properties of physics, on which life depends, are in every respect deliberate. It is, therefore, almost inevitable that our own measure of intelligence must reflect higher intelligence - even to the extreme idealized limit of God."

‘’I had always assumed you to be an atheist,” I said. “But from your last remark I’m not so sure.”

“I’m certainly an atheist in that I reject the deity of the traditional Judeo-Christian religions. I am not a Christian, nor am I likely to become one as far as I can tell. But I think that’s there is a coherent plan in the universe, though I don't know what it's a plan for. In particular, I think that there must have been some ‘intelligence’ behind the emergence of life on Earth. I’m not sure what sort of intelligence this could be, but it would not be supernatural, it would be part of nature in the broad sense. Once we see that life is cosmic it is sensible to suppose that intelligence is cosmic. Now problems of order, such as the sequences of amino acids in the chains which constitute the enzymes and other proteins, are precisely the problems that become easy once a directed intelligence enters the picture. So if one proceeds directly and straightforwardly in this matter, without being deflected by a fear of incurring the wrath of scientific opinion, one arrives at the conclusion that biomaterials with their amazing measure of order must be the outcome of intelligent design. No other possibility I have been able to think of in pondering this issue over quite a long time seems to me to have anything like as high a possibility of being true."

“Professor Hoyle, I think we have arrived at an appropriate point to conclude our discussion. I thank you for a most illuminating exchange.”

“I’ve enjoyed it. Almost as much as the cricket match I was watching when you arrived. And that, I can assure you, is a compliment.”

We both rose, shook hands and I took my leave.

Hermann Weyl

My investigations into the mathematics of the 20th century had led me to the works of a number of outstanding mathematicians. One of these was the German mathematician Hermann Weyl. His work has a vast range, encompassing analysis, algebra, number theory, topology, differential geometry, spacetime theory, quantum mechanics, and the foundations of mathematics. His scientific writing is informed by a rare literary and artistic sensibility—in his words, "Expression and shape mean almost more to me than knowledge itself". He was unusual among scientists and mathematicians of his time in being attracted to idealist philosophy: his idealist leanings can be seen particularly in his work on the foundations of mathematics. In his youth, Kant's doctrines made a great impression on him; later he was stirred both by Fichte's metaphysical idealism and by Husserlian phenomenology. Although Weyl came to question the certainties claimed by idealism, he cleaved always to the primacy of intuition he had first learned from Kant, and to its expression by Fichte as the "inner light" of individual consciousness.

I became intrigued by Weyl's many writings, which display a vast erudition, an acute philosophical awareness, and even, on occasion, a certain playfulness. No matter what the subject may be—mathematics, physics, philosophy—Weyl's writing fascinated me both by the depth of insight it reveals and by its startling departures from academic convention. Who else would have the daring to liken (as he does in the discussion of Space and Time in his great work *Philosophy of Mathematics and Natural Science*), a coordinate system to "the residue of the annihilation of the ego"? Or then (somewhat further on in the same discussion) to express the belief in the impossibility of a completely objective account of individual consciousness by the assertion "...it is shattered by Judas' desperate outcry, 'Why did *I* to be Judas?'"

During his long philosophical voyage Weyl stopped at a number of ports of call: in his youth, Kantianism and positivism; then Husserlian phenomenological idealism; later Brouwerian intuitionism and finally a kind of theological existentialism. But apart from his brief flirtation with positivism (itself, as he says, the result of a disenchantment with Kant's "bondage to Euclidean geometry"), Weyl's philosophical orientation remained in its essence idealist. Nevertheless, while he continued to acknowledge the importance of phenomenology, his remarks in one of his last essays *Insight and Reflection* indicate that he came to regard Husserl's doctrine as lacking in two essential respects: first, it failed to give due recognition to the (construction of) transcendent external world, with which

Weyl, in his capacity as a natural scientist, was concerned; secondly, and perhaps in Weyl's view even more seriously, it failed to engage with the enigma of selfhood: the fact that I am the person I am. Grappling with the first problem led Weyl to identify what he termed "symbolic construction" as providing sole access to objective reality, a position which brought him close to Cassirer in certain respects; while the second problem seems to have led him to existentialism and even, through his reading of Eckhart, to a kind of religious mysticism.

I was determined to meet with this remarkable philosophically minded mathematician. He was a member of the Institute for Advanced Study in Princeton from its establishment in 1933 until he retired in 1951. I decided to try to arrange an interview with him there just before his retirement, choosing to present myself as a senior editor of the *Scientific American.* Accordingly I time-travelled to Princeton in June 1950 and managed to contact him by telephone at his home. He accepted my proposal for an interview and suggested that we meet at his office in the Institute's Fuld Hall the following afternoon.

At the agreed time I entered Fuld Hall, where my attention was drawn to the list of residents' offices displayed in a glass case near the entrance doors. Weyl's office was on the second floor, next to that of the Institute's most celebrated member, Albert Einstein. I knocked on Weyl's office door and was invited inside. As I entered, Weyl, who had been sitting at his desk, rose to greet me, and shook my hand in welcome. I had seen photographs taken of Weyl in his later years, and the cast of his face, broad-browed, kindly and open, had struck me as being entirely consonant with the luminous spirit of his writings. Meeting him in the flesh confirmed these photographic impressions and added something more, for as we shook hands, I fancied I saw in his eyes the gleam of that "inner light of consciousness", which he mentions in nis writings.

There were a pair of armchairs opposite the desk. He invited me to sit down in the nearest one, and settled himself in the other.

I ventured to initiate the conversation by saying: "Professor Weyl, I must thank you for consenting to this interview. As a student and admirer of your works, it is a privilege to engage in discussion with you. I have every hope that our discourse will prove illuminating, not only to the readers of *Scientific American,* but to me, and, perhaps, also to you."

“I am happy to share your hopes, “Weyl responded affably. “Let us attempt to realize them.”

“Thank you. Let me begin by saying that among scientists and mathematicians of the present century you are unusual in being attracted to idealist philosophy. Perhaps among physicists only Erwin Schrödinger - a friend of yours, I believe - shares your philosophical convictions. Would you agree?”

Weyl looked somewhat taken aback, but at the same time not displeased by my question. “Well, Mr. Bellane,” he said,” I’m pleasantly surprised that an interviewer from a scientific magazine should begin his probing with a philosophical question. I’m happy to respond. Yes, first of all, it is quite true that most of my mathematical and scientific contemporaries did not, and do not, share my attraction to philosophy, idealist or otherwise. And yes, my old friend and colleague Schrödinger and I have idealist philosophical leanings. We would both agree with Schopenhauer’s dictum ‘the world is my idea’—that the beginning of all philosophical thought is the realization that the perceptual world is but an image, a vision, a phenomenon of subjective consciousness, that consciousness does not directly grasp a transcendental real world which is as it appears. We would both agree that the only things which are directly given to us, and which we can know completely, are objects of subjective consciousness. In short, we would both agree that the *fons et origo* of the world lies in Consciousness.

“But at this point our philosophical paths start to diverge. Schrödinger has declared his allegiance to the ancient Indian doctrine of Vedantism according to which the multiplicity of minds or consciousnesses is only apparent, that there is truly only one mind, a universal consciousness of which individual minds are just facets. On this view, subjectivity becomes absorbed into the Objective.

“My own view of the world is, by contrast, essentially pluralistic. I understand, and to some extent share the attraction to a monist account of the world - I am a mathematician, for my sins, after all. But I cannot subscribe to the notion that the world is a static One consisting solely of an all-pervading universal consciousness. Rather, I see reality arising from subjective consciousness as a multiplicity formed by three fundamental elements: the Ego or “I”, the Conscious Other, or “Thou”; and the Unconscious Other, the external ‘objective’ world or ‘It’. Now none of these elements is given directly to consciousness; rather, each is arrived at through an act of postulation. These postulations are not conscious judgments, but metaphysical assertions, acts of acknowledgment and belief.

While traditional idealism assumed that the phenomenon of consciousness guarantees the reality of the Ego in an essentially different and somehow more certain manner than the reality of the remaining elements, I believe this to be an error. I am convinced that in the transition from consciousness to reality the Ego, the Thou and the world rise into existence indissolubly connected and, as it were, at one stroke.

"Nevertheless, like planets situated at different distances from a central sun, these elements vary in their degree of closeness to consciousness itself. The Ego is closest, since, given that the existence of consciousness *sui generis* is unquestionable, the Ego is postulated to furnish a first answer to the question: what is it that possesses consciousness? The immediate answer is, of course, *I* possess consciousness. The Ego is postulated by consciousness to be a reification of the *I,* and so the possession of consciousness becomes an attribute of the Ego. It follows that the Ego has direct access to all the raw materials of the existent which are immediately and irreducibly present to subjective consciousness. It is, so to speak, the 'lighted circle of intuition'. And like subjective consciousness with its 'unity', the Ego is immediately confronted with its own uniqueness, the affirmation that there is only one of me. But in an act of self-reflection, through grasping 'what I am for myself', the Ego comes to recognize that it has an objective *function*, namely to be a conscious-existing carrier of the world of phenomena. It is then but a short step for the Ego to transcend its uniqueness through the act of defining an 'ego' to be an entity performing that same function *for itself*. That is, an ego is precisely what I am for myself (in other words, what the Ego is for itself)—again a 'conscious-existing carrier of the world of phenomena' - and yet *other* than myself. 'Thou' is the term the Ego uses to address, and so to identify, an ego in this sense. 'Thou' is thus the Ego generalized. The Ego grasps that it exists within a world of Thous, that is, within a world of other Egos similar to itself. While the Ego has, of necessity, no direct access to any Thou, it can, through analogy and empathy, grasp what it is to be Thou, a conscious being like oneself. As a poet of my acquaintance put it,

To admire, yet not to envy,
To challenge, but never to despise,
To love, perhaps also to pity,
To transcend the singularity of the self,
To see in you the luminosity I see in me—
That self-same consciousness I possess—
The image of the self refracted through itself.
Yet not to require reciprocation,
Bowing to the asymmetry of the I and the Thou.

That is what is required of me –
And so, paradoxically, also of you."

At this point Weyl paused. I took the opportunity to remark: "That is an illuminating account of the importance you ascribe to the individual consciousness, in oneself and others. I was struck by your remark in *The Philosophy of Mathematics and Natural Science* that any belief in the possibility of a completely objective account of individual consciousness is shattered by Judas' desperate outcry, 'Why did *I* have to be Judas?'"

"Ah yes, poor Judas," Weyl said with a sigh. "His tragic predicament serves only too well to personify the enigma of selfhood that emerges once the Ego overcomes its natural solipsism and embraces pluralism by coming to recognize the existence of a world of Thous. The Vedantist picture avoids the problem of selfhood by expanding the solipsism of the Ego into the equally solipsistic monism of the universal consciousness. It also requires one to embrace the doctrine of Maya, that the individual consciousness, the Ego, is a mere illusion, having no more significance than a passing wave on the surface of the cosmic ocean. I don't think this account does justice to the irreducible experience of individual consciousness, and to the existential struggle that individual consciousness must engage in to recognize that it is not unique. Here we are confronted with the metaphysical question concerning the relation between the one pure I of immanent consciousness and the particular lost human being which I find myself to be in a world full of people like me - for example, during the afternoon rush hour on Fifth Avenue in New York! On the one hand, I am a real individual man carrying out real and physical and psychical acts, one among many. On the other hand, I am 'vision' open to reason, a self-penetrating light, immanent sense-giving consciousness, or however you may call it, and as such unique. Nothing is more familiar and revealed to me than this mysterious 'marriage of life and darkness', of self-transparent consciousness and real being that I am myself. This is my knowledge of myself from within, through which I am made aware of my own acts of perception, thought, volition, feeling and doing, all in a manner entirely different from the theoretical knowledge that represents the parallel cerebral process in symbols. The inner awareness of myself is the basis for the more or less intimate understanding of my fellow-men, whom I acknowledge as beings of my own kind. Granted that I do not know of their consciousness in the same manner as my own, nevertheless my 'interpretative' understanding of it is an insight of indisputable accuracy. Its illumining light not only falls on my fellow-men; it also reaches, though with ever-increasing dimness and incertitude, deep into the animal kingdom."

“Could you tell me more,” I asked, about the third component in your metaphysical triad, the external world? How, for example, does the Ego come to gain knowledge of this realm?”

Weyl thought for a moment and responded: “That’s an important question. The relationship of the Ego with the external world, the realm of ‘objective’ reality, is of an entirely different nature from that between the I and the Thou. There is no analogy that the Ego can draw—as it can with the Thou—between itself and the external world, since that world (presumably) lacks consciousness. The external world is radically other, and opaque to the Ego. In particular, while there is compelling evidence that the continued existence of the Ego (one’s consciousness) is dependent on facts about the external world, for instance, the continuing biochemical activity of one’s brain, that dependency cannot be taken as a *primary* datum for the Ego. Like Kant’s noumenal realm, the external world is outside the immediacy of consciousness; it is *transcendent*. The external world simply is, it does not happen. Only to the gaze of my consciousness, crawling along the lifeline of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time.”

Weyl passed briefly and then continued: “The opacity to the Ego of the external world can be partly overcome by constructing a representation of it through the use of *symbols*. I call this procedure *symbolic construction*, or *constructive cognition*. and I regard it as the cornerstone of scientific explanation. “

“Could you outline its essentials ?” I asked.

“I’d be happy to,” Weyl responded. “Symbolic construction, as I conceive it, is carried out in two stages. In the first stage, an immediately given configuration is subjected to variation. One then identifies those features of the configuration that remain unchanged under the variation—the *invariant* features. In the second stage, symbols are introduced in order to split up the judgements in such a way that part of the operations is made independent of the given, and its duration, by being shifted onto the symbols themselves. These symbols are time-resisting and also serve the purpose of preservation and communication. Through a process of reification, the symbols are deemed to be represent properties of an unchanging substrate—the ‘things themselves’. It is precisely the invariance of such features that renders them (as well as the ‘things themselves’) capable of being represented by the ‘time resisting’ symbols introduced in the second stage. As (written) symbols these are communicable without temporal distortion and can be subjected to unrestricted manipulation without degradation. It is the flexibility conferred thereby which enables the use of symbols o be conformable with reality. Nevertheless the symbols are not haphazardly created in response to immediate stimuli, but are introduced, rather, in a structured, yet freely chosen manner which reflects the idea of an underlying order—the 'one real world'—about which not everything is, or can be, known—it is, like the future, 'open towards infinity'. These symbols are embedded into an ordered manifold of possibilities created by free construction and open towards infinity. It is only in this way that we can contrive to predict the future, for the future is not given in actuality. I would add that the reification implicit in the procedure of symbolic construction leads inevitably to its iteration, for the transition from step to step is made necessary by the fact that the objects at one step reveal themselves as manifestations of a higher reality, namely, the reality of the next step. But in the end systematic scientific explanation will reverse the order: first it will erect its symbolic world by itself, without any reference, then, skipping all intermediate steps, try to describe which symbolical configurations correspond to which data of consciousness. In this way the symbolic world becomes identified with the transcendent world- albeit mistakenly. It is symbolic construction which allows us access to the 'objective' world presumed to underpin our immediate perceptions. I would go further to assert that the objective world, being beyond the grasp of intuition, can *only* be presented to us in symbolic form."

I said, "It strikes me that symbolic construction of an objective world beyond the mental, as you have proposed, involves the Ego in two distinct ways. For not only is that world 'constructed' by the Ego, but the materials of construction, the symbols themselves, as signs intended to convey meaning, have no independent existence beyond their graspability by a consciousness. By their very nature these symbols cannot point directly to an external world (even given an unshakable belief in the existence of that world) lying beyond consciousness."

Weyl nodded his approval. "Yes, I agree . It is this fact that points up what I consider to be the fundamental difference between mathematics and physics, the subjects closest to my heart, or my mind at any rate. I believe with Brouwer that mathematics simply lies within the Ego's 'lighted circle of intuition' and so is, in principle at least, completely presentable to that intuition. But the nature of physics is more complicated. To the extent that physics is linked to the transcendent world of objective reality, it cannot arise as the direct object of intuition, but must, like the transcendent world itself, be presented in symbolic form; more exactly, as the result of a process of symbolic construction. This means that physics, unlike mathematics, is not required to behave in an 'intuitive' way. This may serve to explain the highly counterintuitive nature of quantum theory. Indeed, the claims of numerous physicists that the quantum world is accessible to us only through abstract mathematical description provides a vindication of this view."

"With your leave, Professor Weyl," I said, "I'd like now to direct our discussion to the foundations of mathematics, a topic I know has played a large role in your thinking,"

"By all means."

"First of all," I said, "could you characterize what you see as the essence of the mathematical method?"

Weyl responded, "A number of prominent mathematicians, for example G. H. Hardy, hold the view that mathematical reality lies outside us, and that the mathematician's function is to discover or observe that reality. Hardy goes further when he says that the theorems which mathematicians prove are simply the notes of their observations. This is essentially to treat mathematical entities as Platonic Forms, independently existing eternal objects constituting a higher reality. In this Platonic world, the infinite sequence of natural numbers, for example, is conceived of as a completed whole, a closed realm of things in themselves."

"My conception of the nature of mathematics is very different," he went on. I think that mathematics originates not through observation of a world outside us, but arises rather through constructive enlargement of what is given to the mind in intuition. Again take the case of the natural numbers. We see immediately that, given any number, we can always form an immediate successor by adding the number 1 to it. This process generates the sequence of all possible numbers, into which concrete individual numbers such as 2, 3, and 10001 can be embedded. Here the existent is projected into the background of the possible, the background of a manifold of possibilities which is produced and ordered according to a fixed process but is open to infinity. This open manifold of possibilities in no way depends on the actual existence of infinitely many objects in the real world. It is obtained by an a priori process of construction. I see the mathematical method in general as the a priori construction of the possible in opposition to the a posteriori description of what is actually given. "

“What do you regard as the ultimate foundation of mathematical knowledge?” I ventured to ask.

“I believe that intuition, or insight—rather than proof— furnishes the ultimate foundation of mathematical knowledge. This view is not shared by most mathematicians. For example, Dedekind has stated in no uncertain terms that in science, whatever is provable must not be believed without proof. This remark is certainly characteristic of the way most mathematicians think, but in my view it is a preposterous principle. As if such an indirect concatenation of grounds, call it a proof though we may, can awaken any ‘belief’ apart from assuring ourselves through immediate insight that each individual step is correct! In all cases, this process of confirmation—and not the proof—remains the ultimate source from which knowledge derives its authority; it is the ‘experience of truth’.”

“What role do you think the infinite plays in mathematics?” I asked.

“Ah, yes, the infinite,” Weyl responded with a smile, “that elusive idea that has haunted mathematics, and philosophy, from the very beginning. Well , so far as mathematics is concerned, I do not think that an actual infinity can be regarded as given, nor can it be constructed in the mathematical sense. No one can describe an infinite set other than by indicating properties characteristic of the elements of the set. The notion that an infinite set is a ‘gathering’ brought together by infinitely many individual arbitrary acts of selection, assembled and then surveyed as a whole by consciousness, is nonsensical; ‘inexhaustibility’ is essential to the infinite. Nevertheless, the demand for totality and the metaphysical belief in reality inevitably compel the mind to represent the infinite as closed being through a process of symbolic construction. The conception of the completed infinite, even if nonsensical, is inescapable. “

“The very inescapability of the concept of an infinite set was precisely Cantor’s conception,” I said.

“Yes,” Weyl rejoined, “but in creating the realm of sets called by Hilbert ‘Cantor’s Paradise’, Cantor threw off every restraint, using the concept of set with complete freedom, especially when he allowed the construction of the set of all subsets of a given set. Here, one first struck real contradictions on the outermost borders of set theory. Their source can only be uncovered as the daring act already committed from the beginning of mathematics: the treatment of a field of constructive possibilities was treated as a closed realm of things existing in themselves."

"Allow me to return to symbolic construction, "I said. "You have mentioned the use of symbolic construction in connection with the infinite sets of mathematics. It seems to me that the sort of symbolic construction involved in this case must be different from its use in getting to grips with the external 'objective' world. For presumably the external world consists of 'things in themselves' which have real existence, even if we can only represent them symbolically. But the completed infinities of mathematics, as you have pointed out, are 'nonsensical', and so lack not only real, but even possible existence. Yet in set theory symbols are routinely introduced to denote (completed) infinite sets such as the sets of natural numbers and real numbers. Here symbols are being employed to denote entities which can have no possibility of existing, in a literal sense. Would you care to comment on this?"

"That's a very good point," Weyl responded. "Perhaps it would be better to describe the employment of symbols in set theory to denote infinite sets as symbolic formalization rather than symbolic construction. Hilbert introduced a strict form of symbolic formalization in order to investigate the consistency of mathematics. In his version of symbolic formalization the symbols introduced are deprived of all meaning and the investigation takes place at a purely syntactic level, much like grammatical analysis in linguistics. But the symbolic formalization underlying axiomatic set theory is of a different sort. Here the symbols are treated as if they have a meaning, even if, in the case of symbols intended to denote infinite sets, that meaning, if taken literally, is nonsensical. But in operating within the theory itself this is set aside and while the symbols have a formal character, they are still, in an informal sense, taken as if they refer to actual (infinite) sets. This is analogous to the situation in Euclidean geometry, in which the straight lines and circles are treated as being perfect, even though no such perfect entities exist in nature. Of course in the case of set theory there is also considerable risk of outright inconsistency arising, even in the axiomatic systems devised by Zermelo and others which avoid the evident contradictions in Cantor's vision."

"Even if one tolerates set theory's reification of the actual infinite," Weyl continued, "another, perhaps even more serious difficulty, arises, as emphasized by Poincaré and Russell. This is the fact that allowing the formation of the set of all subsets of a given set enables the introduction of sets given by means of definitions having a circular character. Logicians call such definitions 'impredicative'. The hallmark of an impredicative definition of an object is that it involves a reference to a set of objects which contains the object being defined. The circularity of a definition of this sort should be evident. Of course, the use of impredicative definitions does not automatically lead to contradictions, indeed is often harmless."

"'The brightest star in the sky' is a good example," I interjected.

Weyl smiled. "Yes indeed, but of course in that case the set of stars in the sky, being finite, is given in advance. Impredicative definitions in set theory typically involve the set of all subsets - the power set - of an infinite set, for example the set of natural numbers and it is by no means clear how one is to ascribe a clear meaning to such entities: at the very least, one would have to pin down the meaning of 'arbitrary subset'. Since the development of mathematical analysis within set theory depends on the use of the power set and impredicative definitions, I cannot regard set theory as providing a secure foundation for mathematics."

I said, "You have devoted a great deal of attention to the problem of how to construct the mathematical continuum, as evidenced in your 1918 book *Das Komtinuum.* Would you describe the development of your ideas in this area?"

"By all means. By 1918 I had come to believe that, the work of Cauchy, Weierstrass, Dedekind and Cantor notwithstanding, mathematical analysis would not bear logical scrutiny. I went so far as to make the melodramatic declaration that its essential concepts and procedures involved vicious circles to such an extent that every cell of that mighty organism is permeated by contradiction. In writing *Das Kontinuum* my goal was to formulate an exact account of the continuum of real numbers free of the circularities in the set-theoretic account arising from the use of impredicatve definitions, but at the same time avoiding the complications and doubtful assumptions of Russell and Whitehead's ramified types. As I saw it, there is an unbridgeable gap between intuitively given continua (e.g. those of space, time and motion) on the one hand, and the 'discrete' exact concepts of mathematics (e.g. that of real number) on the other. I was convinced that this fact meant that the construction of the mathematical continuum could not simply be 'read off' from intuition. I felt, rather, that the mathematical continuum must be treated as if it were an element of the transcendent realm, and so, in the end, justified in the same way as a physical theory. For me, it was not enough that the mathematical theory of the continuum be consistent; it must also be reasonable. I was well aware of the difficulty of achieving these goals, above all because the conceptual world of mathematics is so foreign to what the intuitive continuum presents to us that it would be absurd to demand that the two coincide. Moreover, the continuity in the flow of time and of motion given to us immediately in intuition eludes mathematical presentation as a totality of discrete 'stages' in accordance with that part of its content which can be conceptualized in an exact way. Exact time- or space-points are not the ultimate, underlying atomic elements of the duration or extension given to us in experience. I agreed with Brentano that the continuum concept is derived from primitive sensible intuition and indeed that all our sensible intuitions present us with that which is continuous. Nevertheless, I also accepted that only reason, which thoroughly penetrates what is experientially given, is able to furnish an exact account of the continuum concept. And this exactness itself, I was convinced, can only be found in the arithmetico-analytic concept of the real number. But I could not banish the uncomfortable feeling that once our subjective experience has been turned into a real process in a real world and our phenomenal time has spread itself out over this world, we would still fail to be satisfied with replacing the continuum by the exact concept of the real number, in spite of the essential and undeniable inexactness arising from what is actually given in consciousness.

"I came to see that I could not provide a mathematical formulation of the continuum as it is presented to intuition. My goal became, first, to achieve consistency by putting the arithmetical notion of real number on a firm logical basis, and then to show that the resulting theory is reasonable by employing it as the foundation for a plausible account of continuous process in the objective physical world. In *Das Kontinuum* I tried to furnish analysis with a predicative formulation—not, as Russell and Whitehead had attempted, by introducing a hierarchy of logically ramified types, which seemed to me far too complicated—but rather by confining the principle of set formation to sets definable by formulas whose bound variables range over just the initial given entities, that is, natural numbers. This amounts to restricting analysis to what can be done in terms of natural numbers with the aid of three basic logical operations, together with the operation of substitution and the process of primitive recursion. I was aware that the effect of this restriction would be to render unprovable many of the central results of classical analysis—e.g., Dirichlet's principle that any bounded set of real numbers has a least upper bound—but l was prepared to accept this as part of the price that must be paid for the security of mathematics. "

''Didn't you make a bet with George Pólya in connection with all this?", I ventured to ask.

Weyl shrugged. "Yes, in Zurich in February 1918 I rashly made a bet with Pólya in the presence of twelve witnesses , all of whom were mathematicians, that within 20 years, Pólya, or a majority of leading mathematicians, would come to recognize the falsity of the least upper bound property. When the bet was eventually called, everyone— with the single exception of Gödel—agreed that Pólya had won."

"Your making this bet," I said, "shows your confidence at the time that classical mathematics would have to undergo drastic modifications. Even so, you cannot have been entirely happy with this fact, since it placed such severe constraints on the practice of mathematics."

"That is true," Weyl responded. "And I also came to be displeased with the 'atomistic' or 'discrete' character of the continuum I had constructed. I felt that it simply did not do justice to the essential and irreducible continuity, the cohesion, of a true continuum. I welcomed Brouwer's intuitionistic account of the continuum as a medium of free Becoming in which real numbers are identified as sequences generated by free acts of choice, and open to infinity. Brouwer's continuum does not dissolve into a set of real numbers as finished entities. It seemed to me that Brouwer had come closer than anyone else to bridging that unbridgeable chasm between the intuitive and mathematical continua. In particular, I found compelling the fact that the Brouwerian continuum is not the union of two disjoint nonempty parts - that it is, so to speak, indecomposable. I had long felt that a genuine continuum cannot be divided into separate fragments, that, as Anaxagoras claimed, a continuum defies the chopping off of its parts with a hatchet. I also agreed with Brouwer that all properly defined functions on a continuum are continuous."

"So in essence you may be said to have joined Brouwer in your view of the continuum. But of course it remained the case that many of the central theorems of classical analysis are not provable in intuitionistic mathematics."

"That's right," Weyl acknowledged ruefully. "I felt that Brouwer had developed the beginnings of analysis in an appealingly natural manner, all the time preserving the contact with intuition much more closely than had been done before. It seemed to me that mathematics with Brouwer had gained its highest intuitive clarity. But I could not escape the fact that this gain in intuitive clarity had been bought at a considerable price. For in advancing to higher and more general theories the inapplicability of the simple laws of classical logic eventually results in what I saw as an almost unbearable awkwardness.

And the mathematician would watch with pain the greater part of his towering edifice he believed to be built of concrete blocks dissolve into mist before his eyes."

"Your alliance with Brouwer cannot have pleased Hilbert!" I exclaimed.

"It certainly did not!" said Weyl. "I recall with amusement, and, I have to admit, a certain chagrin, that when I hailed Brouwer's intuitionistic mathematics as a revolution Hilbert responded by calling it a mere *putsch*. What he found particularly objectionable about Brouwer's approach was its rejection of certain central laws of classical logic such as the law of excluded middle and the law of double negation. He saw this as an intolerable restriction on the activity of mathematicians, likening the denial to the mathematician of the use of the law of excluded middle to the denial to the astronomer of the use of the telescope or the use of the fists to the boxer. Yet he was also aware that intuitionism's questioning of the law of excluded middle arose from its strict analysis of the meaning of existential statements in mathematics, in which the truth of a such a statement does not necessarily follow from the fact that its denial leads to a contradiction. In his later attempt to establish the consistency of classical mathematics, he accepted the intuitionistic meaning of existential statements, and attempted to show that the classical rules governing them are at least consistent."

I said: "Hilbert was very concerned to defend classical mathematics against what he saw as Brouwer's destructive attacks, was he not?"

"Yes, indeed he was, passionately so. His response was to develop an entirely new approach to the foundations of mathematics with the ultimate goal of establishing beyond doubt the consistency of the whole of classical mathematics, including arithmetic, analysis, and Cantorian set theory. He believed that attaining that goal would place classical mathematics securely beyond what he saw as the destructive reach of the intuitionists. Perhaps I can outline his approach, which came to be known as Hilbert's program. Its core was the translation of the whole apparatus of classical mathematical demonstration into a simple, finitistic framework , which he called 'metamathematics', involving nothing more, in principle, than the straightforward manipulation of symbols, taken in a purely formal sense, and devoid of further meaning. Within metamathematics itself, he imposed a standard of demonstrative evidence stricter even than that demanded by the intuitionists, a form of finitism rivalling, ironically, that of Kronecker, which he has fought against in his youth. The demonstration of the consistency of classical mathematics was to be achieved by showing, within the constraints of strict finitistic evidence he insisted on, that the formal metamathematical counterpart of a classical proof in that system can never lead to an assertion evidently false, such as 0=1. The establishing of this weas the essence of Hilbert's Program. The fact that Hilhert`s Program involves the study of the *form* of an axiomatic theory as opposed to its *meaning* led to it being identified as *formalism.*

"Hilbert considered that, in the last analysis, the only completely reliable, irreducibly self-evident constituents of mathematics are *finitistic,* that is, involve just the finite manipulation of surveyable domains of concrete objects, in particular, mathematical symbols presented as marks on paper. Hilbert may be seen as following Kant in attempting to ground mathematics on the apprehension of spatiotemporal configurations; but Hilbert restricted these configurations to concrete signs.

"Hilbert regarded consistency as the touchstone of existence, and so for him the important thing was the fact that no inconsistencies can arise within the realm of concrete signs, since correct descriptions of concrete objects are always mutually compatible. In particular, within the realm of concrete signs, actual infinity cannot generate inconsistencies since, again along with Kant, he held that this concept cannot correspond to any concrete object. Hilbert's view was that the formal soundness of mathematics issues ultimately, not from a *logical* source, but from a *concrete* one[, in much the same way as the consistency of truly reported empirical statements is guaranteed by the concreteness of the external world – that there are, so to speak, no contradictions in nature. Hilbert's program can be seen as an attempt to provide mathematics with a new foundation - not by reducing it to logic, but instead by *representing its essential form within the realm of concrete symbols.*

"Hilbert divided mathematical propositions into two categories: those that refer only to concrete, finite objects in the above sense he termed *real*, *concrete*, or *contentual* propositions, and all other mathematical propositions he termed *ideal* propositions. (Thus, for example, an arithmetic proposition such as 2+2=4 would count as a real proposition, while *there exists an odd perfect number* would count as an ideal one· Hilbert viewed ideal propositions as akin to the ideal lines and points "at infinity" of projective geometry. Just as the use of these does not violate any truths of the "concrete" geometry of the usual Cartesian plane, so he hoped to show that the use of ideal propositions—even those of Cantorian set theory—would never lead to falsehoods among the real propositions, that such use *would never contradict any self-evident fact about concrete objects*. That is, *classical mathematics is concretely or finitistically sound.* Establishing the soundness of classical mathematics in this sense by strictly concrete, and so unimpeachable means was the central aim of Hilbert's program. With the demonstration of this soundness, mathematicians would be free to roam unconstrained within 'Cantor's Paradise' - that is, to continue to use with confidence the infinitistic mathematics of Cantor and others which had enabled mathematics to make such spectacular strides. All this was to be achieved by setting out a classical mathematical theory as a purely formal system of symbols, devoid of meaning, and then showing that no proof in the system can lead to a false assertion, e.g. 0 = 1. This, in turn, was to be done by employing the *metamathematical* technique of replacing each abstract classical - ideal - proof of a real proposition by a concrete, finitistic proof. Since, plainly, there can be no concrete proof of the real proposition 0 = 1, there can be no classical proof of this proposition either, and so classical mathematical reasoning is consistent."

"The significance and scope of Hilbert's programme must have struck you immediately," I said. "But did you think that it could be successfully carried out, and were you entirely happy with its goal?"

"Well, Weyl responded. "I was indeed impressed with the audacity of Hilbert's program. But whether that program could be successfully carried out was, of course, still an open question, even though Hilbert himself was at the time of its formulation was entirely convinced that it could. But independently of this issue I was concerned about what I saw as the loss of content resulting from Hilbert's thoroughgoing formalization of mathematics. I strongly believed that if mathematics were to remain a serious cultural concern, then some *sense* must be attached to Hilbert's game of formulae. I thought that this sense could only be supplied by fusing mathematics and physics so that the mathematical concepts of number, function, and Hilbert's symbols generally partake in the theoretical construction of reality in the same way as do the physical concepts of energy, gravitation, electron, etc. Of course, just as in theoretical physics, Hilbert's account of mathematics, with its ideal assumptions , already goes beyond the bounds of intuitively ascertainable states of affairs. If Hilbert's realm of contentual or 'real' propositions—the domain of metamathematics—corresponds to that part of the world directly accessible to insight or phenomenal knowledge, then 'serious' mathematics—the mathematics that practicing mathematicians are actually engaged in doing—corresponds to Hilbert's realm of 'ideal' propositions. I regarded this realm as the counterpart of the domain generated by symbolic construction, the transcendent world focussed on by theoretical physics."

"So it would be fair to say that you viewed Hilbert's approach, as being in essence, a symbolic representation of the transcendent mathematical realm."

"Yes, and I therefore regarded Hilbert's approach as a natural development. Philosophically speaking, it might be said that if the set-theoretical approach is the stage of naive realism which is unaware of the transition from the given to the transcendent, then Brouwer represents idealism, by demanding the reduction of all truth to the intuitively given. In Hilbert's formalism, finally, consciousness makes the attempt to 'jump over its own shadow', to leave behind the stuff of the given, to represent the transcendent— inevitably through the symbol. I later came to feel that Hilbert's doctrine was beginning to prevail over intuitionism. This in turn struck me as foreshadowing a decisive defeat of the philosophical attitude of pure phenomenology, which had, through Husserl's influence, played a large part in my philosophical formation. Phenomenology now seemed to me insufficient for the understanding of creative science even in the area of cognition that is most primal and most readily open to evidence, that is, mathematics. By this time I had become convinced that creative science must necessarily transcend what is phenomenologically given, and I already accepted that pure phenomenology is incapable of accounting for theoretical physics, let alone the whole of existence. But I confess found it painful to concede the analogous claim in the case of mathematics. I had believed that in mathematics, taken by itself, one should restrict oneself with Brouwer to the intuitively cognizable truths, that nothing compels us to go farther. If mathematics could be taken by itself, then there would be no need for it to justify its practices by resorting to symbolic construction, to employ symbols which in themselves signify nothing - nothing, at least, accessible to intuition. But I finally had to come to terms with the idea that mathematics could not simply be taken by itself, that it has a larger role to play in the world beyond its service as a paradigm, however pure, of subjective certainty."

"What was your reaction to Gödel's incompleteness theorems and what do you think their impact has been on Hilbert's program? In particular, did they cause you to rethink the significance of Hilbert's program?"

"Well, Gödel has left us little hope that a formalism wide enough to encompass classical mathematics will be supported by a proof of consistency. While Brouwer has made clear to us to what extent the intuitively certain falls short of the mathematically provable, Gödel shows conversely to what extent the intuitively certain goes beyond what (in an arbitrary but fixed formalism) is capable of mathematical proof. Nevertheless , Gödel's work did not affect my fundamental understanding of Hilbert's metamathematics as an attempt to represent the transcendent through symbols. While Gödel's remarkable discovery that there is no acceptable way of demonstrating the consistency of such representations shows that the principal aim of Hilbert's program is unrealizable, it by no means invalidates the idea of pursuing further investigations of the formal properties of such representations. In any case it seems that Gödel himself has claimed that the program for establishing the consistency of arithmetic might be salvageable through an enlargement of the domain of objects admitted into finitistic metamathematics. That is, by allowing finite manipulations of suitably chosen abstract objects in addition to the concrete ones Gödel hopes to strengthen finitistic metamathematics sufficiently to enable the consistency of arithmetic to be demonstrable within it. I do not know how far he has developed these ideas, but there is every hope that he will be successful."

"How do you see the present situation in the foundations of mathematics?" I ventured to ask.

"I see the emergence of a renewed interest in axiomatic systems developed before Hilbert without Hilbert's ambitious dreams, for example Zermelo's set theory, Russell's and Whitehead's ramified type theory and Hilbert's own axiom systems for geometry. I think of the battle between Hilbertian formalism and Brouwerian intuitionism in which I participated in the 1920s as having given way to a dextrous blending of the axiomatic approach to mathematics championed by Bourbaki and the algebraists - themselves mathematical descendants of Hilbert - with constructive procedures associated with geometry and topology. Nevertheless, the ultimate foundations and the ultimate meaning of mathematics remain an open problem; we do not know in what direction it will find its solution, nor even whether a final objective answer can be expected at all. Mathematizing may well be a creative activity of man, like music, the products of which not only in form but also in substance defy complete objective rationalization. One thing is clear: we are less certain than ever about the ultimate foundations of logic and mathematics; like everybody and everything in the world today, we have our 'crisis'. We have had it for nearly fifty years. Outwardly it does not seem to hamper our daily work, and yet I for one confess that it has had a considerable practical influence on my mathematical life: it directed my interests to fields I considered relatively safe and it has been a constant drain on my enthusiasm and determination with which I pursued my research work. The experience is probably shared by other mathematicians who are not indifferent to what their scientific endeavours mean in the contexts of man's whole caring and knowing, suffering and creative existence in the world."

“Professor Weyl,” I said, “Mathematics has been called the science of the infinite. Indeed, the mathematician invents finite constructions by which questions are decided which by their very nature refer to the infinite. But before the invention of set theory and transfinite numbers this reference was very indirect. In fact the infinite did not play an active roe in mathematics at all. Rather, it functioned as a kind of ever-receding, unattainable horizon on the mathematical landscape. Actually it was the concept of the *infinitesimally small,* rather than the infinitely large, that entered mathematics from the time of the ancient Greeks and later came to play a central role in the development of the calculus, mathematical analysis and mathematical physics. Would you comment on this?”

Weyl smiled. “Ah yes, the infinitesimal, that elusive creature! The idea of the infinitesimal emerged from the attempt to conceive of the continuum as Being in itself. In fact there have been three such attempts in the history of thought. In the first and most radical of these attempts the continuum is reduced to countably many discrete elements, atoms. With regard to matter, this path, as the doctrine of atomism, was initiated by Democritus in antiquity, and followed by Epicurus, Plato, the early Islamic philosophers, Giordano Bruno, Galileo, Cavalieri and Hume. Atomism reached its zenith with its brilliant successes in physics and chemistry in the 19th century, and stimulated by quantum theory the idea again arises today in discussions about the foundations of physics. But so far the idea of reducing the continuum, whether that of geometry, space, time or motion, to discreetness, has, in my view, always remained mere speculation and has never achieved sufficient contact with reality.

“The infinitesimal itself embodies the second attempt at conceiving the continuum as pure Being. The idea of an infinitesimal magnitude again dates back to the ancient Greeks, arising in connection with their early efforts at quadrature of the circle by regarded it as a regular polygon composed of an unbounded number of infinitesimal straight lines. In this way an infinitesimal magnitude is as a continuum ‘viewed in the small’. They reappear as so-called ‘indivisibles’ in the mathematics of the late Middle Ages and the 16th century. In the development of the differential calculus in the 17th and 18th centuries the use of infinitesimal magnitudes, for example in the form of tiny area or volume elements, was indispensable. An essential property of these infinitesimal magnitudes was that they could be regarded as possessing only elementary geometric properties. This period also the emergence of the idea of an *infinitesimal quantity*, or number, that is, is one which, while not coinciding with zero, is less than any finite number. Another conception of an infinitesimal was that of a quantity is so small that its square and all higher powers can be set to zero. But their dubious logical status led in the nineteenth century to their abandonment and replacement by the limit concept, with an infinitesimal being conceived, following Newton, as a variable quantity approaching, but not actually becoming, zero. "

"And so," I interjected, "in its struggle with the infinitely small, you see the limiting process as winning a final victory."

" Yes, I do," Weyl responded. "Its logical foundations are free of the obscurities of the infinitesimal and indeed renders the very idea of the infinitely small superfluous. Infinitesimal analysis proposed drawing conclusions by integration from the behavior in the infinitely small, which is governed, as I have said, by elementary laws. But if the infinitely small is not interpreted only 'potentially', here, in the sense of the limiting process, then the one has nothing to do with the other, the processes in infinitesimal and in finite realms become independent of each other, the link which binds them together is severed. Newton and Leibniz seemed to have the correct view, which they formulated more or less clearly, that the infinitesimal calculus is concerned with the approach to zero by a limiting process. But they lacked the ultimate insight that the limiting process serves not only to determine the value of the limit but also to establish its existence. The third attempt to 'save' the continuum in the Platonic sense may be seen in the modern set-theoretic foundations of analysis. Here, through the use of the completed infinite, the apparent 'Becoming' of the limit process is replaced by the 'Being' of the limit itself."

"Professor Weyl," I observed, "you have said that if the infinitely small is not interpreted 'potentially', in the sense of the limiting process, then the processes in the infinitesimal and the finite realms become independent of each other, the link which binds them together is severed. What if it were possible to reforge this link by formulating a suitably rigorous notion of infinitesimal?"

Weyl gave me a quizzical look. "That is an intriguing suggestion. Do you have a proposal in mind?"

"Well, yes, I do. It's rather sketchy but you might find it of interest. Would you permit me to outline it?"

"By all means, go ahead"

“You may recall Hilbert’s 1917 paper ‘Axiomatic Thinking’ in which he touches on the axiomatic treatment of continuity and, as he puts it, "the dependence of the propositions of a field of knowledge on the axiom of continuity. In order to formulate an exact principle of continuity Hilbert turns to physics, and he states then proceeds to state it as follows: ‘If a certain arbitrary degree of exactitude is prescribed for the validity of a physical assertion, a small range shall then be specified, within which the presuppositions prepared for the assertion may freely vary so that the deviation from the assertion does not overstep the prescribed degree of exactitude.’ Let me call this the physical continuity axiom , PCA for short. Evidently this is an empirical version of the familiar (ε, δ) definition of a continuous function. More precisely, the axiom asserts that any function from real numbers to real numbers associated with a physical assertion is (ε, δ) - continuous. This is an updated version of Leibniz``s Principle of Continuity: *Natura non facit saltus.*

“Now,” I went on, “before the 19th century PCA would have been formulated in terms of infinitesimals, perhaps as follows: ‘If the degree of exactitude is prescribed for the validity of a physical assertion, is prescribed to be within infinitesimal limits, then also within infinitesimal limits the presuppositions prepared for the assertion may freely vary so that the deviation from the assertion does not overstep the prescribed infinitesimal limits’. Let me call this the Principle of Infinitesimal Continuity, PIC for short): any real function sends infinitesimally close points to infinitesimally close points.”

I paused, and gestured towards the large blackboard mounted on one of the office walls. “May I use the blackboard to chalk some symbols?”

“By all means.”

I rose and stood at the blackboard. “Hilbert’s continuity axiom was formulated for the physical realm, but it can be extended to mathematics where it takes the form of a principle you have already mentioned in passing, namely, Brouwer’s continuity principle BCP for short. All functions from real numbers to real numbers are continuous. The question of the consistency of this strengthened principle of continuity arises immediately. It might seem at first glance that BCP is inconsistent since the “blip” function b: $\mathbb{R} \rightarrow \mathbb{R}$ defined by $b(0) = 1$, $b(x) = 0$ for $x \neq 0$ is obviously discontinuous. But the condition that b is defined on the whole of $\mathbb{R}$ rests on the unquestioned assumption that, for any real number x , either $x = 0$ or $x \neq 0$. This in turn rests on the Law of Excluded Middle , LEM for short – the logical principle, going back to Aristotle, that, for any proposition, either it or its negation must be true. While LEM is a core principle of classical logic, it is, as you well know, not affirmed in intuitionistic logic, the system of logic implicit in Brouwer's conception of mathematics and later made explicit by his student Heyting. Thus, while BCP is inconsistent with classical mathematics, that is, mathematics based on classical logic, it appears to be consistent with intuitionistic mathematics. It is easily seen that, within intuitionistic mathematics, LEM is refutable from BCP in the sense that BCP $\Rightarrow \neg\forall x{\in}\mathbb{R}\ (x = 0 \vee x \neq 0)$. Here we have an example of a mathematical axiom actually refuting a logical axiom.

"An even stronger version of BCP (implicitly adhered to in differential geometry) is: SP All functions from reals to reals are smooth, i.e. arbitrarily many times differentiable. (More generally, all functions between manifolds are smooth). Axiom SP characterizes what should naturally be called 'Smooth Analysis'."

At this point Weyl said: "Axiom SP looks to me more like a theorem than an axiom. Can it be derived from simpler principles?"

"Yes indeed," I replied, "and those principles involves infinitesimals defined in a manner that you have already mentioned. The theory based on these principles is called Smooth Infinitesimal Analysis , **SIA** for short. With your leave, I will present the essentials of **SIA** in the form of a brief lecture. Is this agreeable to you?"

"This 'interview' , returned Weyl, "has indeed taken a surprising turn . But I'm intrigued, so proceed with your lecture."

"You are most gracious."

I then proceeded to give the following impromptu lecture, which I record here.

In **SIA** we suppose given a subset Δ of the set **R** of real numbers called the *domain of infinitesimals.* Δ is assumed to satisfy the following conditions:

(1) $0 \in \Delta$

(2) *Any multiple of an infinitesimal is an infinitesimal,* that is, for any $a \in \mathbf{R}$, $x \in \Delta \Rightarrow ax \in \Delta$. It follows immediately from this that Δ is *symmetric around* 0, that is, $\forall x\ (x \in \Delta \Leftrightarrow -x \in \Delta)$.

(3) The ***Affineness Principle***. This asserts that all maps $\Delta \to \mathbf{R}$ are *affine* in the following strong sense:

- for any f: $\Delta \to \mathbf{R}$ there is a *unique* $a \in \mathbf{R}$ such that, for all $\varepsilon \in \Delta$, $f(\varepsilon) = f(0) + \varepsilon a$. This real number a is called the *slope* of f, and is written slp(f). Thus, for any $f : \Delta \to \mathbf{R}$ and all $\varepsilon \in \Delta$, $f(\varepsilon) = f(0) + \varepsilon\, \mathrm{slp}(f)$.

Given $f : \Delta \to \mathbf{R}$, let $f^* : \Delta \to \mathbf{R} \times \mathbf{R}$ defined by $f^*(\varepsilon) = (\varepsilon, f(\varepsilon))$. f^* may be thought of as the graph of the map f n the plane. The Affineness Principle can be seen as asserting that this graph is a straight line with slope slp(f) passing through $(0, f(0))$. Thus the effect of any map on Δ is to translate and rotate it; in effect Δ behaves like a short "rigid rod", just long enough to have a slope, but too short to bend. It is, as it were, a geometric object possessing location and direction but lacking extension.

The Affineness Principle has two immediate consequences:

- **The nondegeneracy of Δ :** $\Delta \neq \{0\}$. For suppose $\Delta = \{0$, and let a, b be any unequal real numbers. Then the two maps $\varepsilon \mapsto \varepsilon a$ and $\varepsilon \mapsto \varepsilon b$ would both be identically 0, contradicting the Affineness Principle.
- **The Cancellation Principle:** for any $a, b \in \mathbf{R}$, if $\varepsilon a = \varepsilon b$ for all $\varepsilon \in \Delta$, then $a = b$. This follows immediately from the uniqueness condition in the Affineness Principle.

I can now show that Δ consists of *nilsquare* quantities, that is,

$$\Delta \subseteq \{x \in \mathbf{R}: x^2 = 0\}.$$

To prove this, write s: $\Delta \to \mathbf{R}$ for the map $\varepsilon \mapsto \varepsilon^2$. Then we have, for $\varepsilon \in \Delta$,

$$\varepsilon^2 = s(\varepsilon) = s(0) + \varepsilon\, \mathrm{slp}(s) = \varepsilon\, \mathrm{slp}(s)$$

and, since $-\varepsilon \in \Delta$,

$$\varepsilon^2 = (-\varepsilon)^2 = s(-\varepsilon) = s(0) - \varepsilon\, \mathrm{slp}(s) = -\varepsilon\, \mathrm{slp}(s).$$

Hence $\varepsilon^2 = -\varepsilon^2$, so that $\varepsilon^2 = 0$.

Notice that I do not claim that Δ comprises *all* nilsquare quantities. This is not needed.

Let us call a *real function* any real-valued function defined on an interval in to **R.** It is assumed ethat intervals are closed under the addition of infinitesimals. The derivative of an arbitrary real function can now be introduced. Given an interval **I** in **R** and a function

$f : \mathbf{I} \to \mathbf{R},$ for each $x \in \mathbf{I}$ define the function f_x: $\Delta \to \mathbf{R}$ by $f_x(\varepsilon) = f(x + \varepsilon)$. The *derivative* $f' : \mathbf{I} \to \mathbf{R}$ of f is defined by $f'(x) = \text{slp}\ (f_x)$. It follows easily that

$$f(x + \varepsilon) = f(x) + \varepsilon f'(x).$$

This is the *Fundamental Equation of the Differential Calculus* in **SIA**. The quantity $f'(x)$ is the slope at x of the curve determined by f and the infinitesimal

$$\varepsilon f'(x) = f(x + \varepsilon) - f(x)$$

is the infinitesimal change or *increment* in the value of f on passing from x to $x + \varepsilon$.

Derivatives of elementary functions are easily calculated in **SIA** using the Cancellation Principle. For example, here is the calculation of the derivative of the function x^n:

$$\varepsilon(x^n)' = (x + \varepsilon)^n - x^n = \varepsilon n\, x^{n-1} + \text{terms in } \varepsilon^{2\text{-}} \text{ and higher powers} = \varepsilon n\, x^{n-1}$$

Hence, by the Cancellation Principle,

$$(x^n)' = n\, x^{n-1}.$$

And here is the calculation of the derivative of the function $1/x$ (for $x > 0$):

$$\varepsilon(1/x)' = 1/x{+}\varepsilon - 1/x = -\varepsilon/x(x{+}\varepsilon) = -\varepsilon(x{-}\varepsilon)/x(x{+}\varepsilon)(x{-}\varepsilon)$$

$$= -\varepsilon x{+}\varepsilon^2/x(x^2{-}\varepsilon^2)$$

$$= -\varepsilon x/x^3$$

$$= -\varepsilon/x^2.$$

Cancelling ε on both sides of the equation gives

$$(1/x)' = -1/x^2.$$

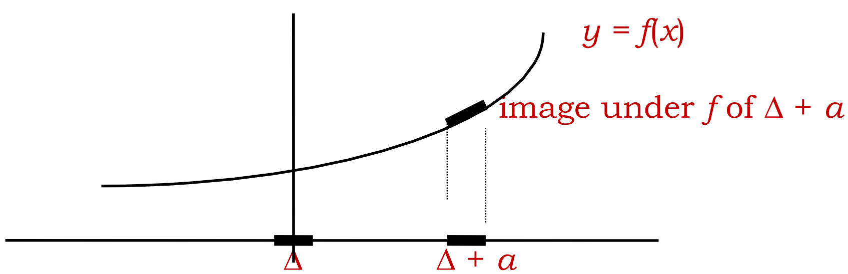
From the Fundamental Equation a version of the *Principle of Continity* can be deduced, namely, that in **SIA** *all real functions are continuous*, in the sense of *sending neighbouring points to neighbouring points.* Here two points x, y on $\mathbf{R}$ are said to be *neighbours* if $x - y$ is in $\mathbf{\Delta}$, that is, if x and y differ by an infinitesimal. To see this, given a real function f and neighbouring points x, y, note that $y = x + \varepsilon$ with ε in $\mathbf{\Delta}$, so that

$$f(y) - f(x) = f(x + \varepsilon) - f(x) = \varepsilon f'(x).$$

Since $\varepsilon f'(x)$ is infinitesimal, the result follows.

From this it can be seen that in **SIA** *every real function is differentiable, hence continuous.* In particular the derivative of an arbitrary real function is itself differentiable, so that any real function is arbitrarily many times differentiable. This fact justifies the use of the term "smooth".

If one thinks of a real function *f* as defining a curve, then, for any *a*, the image under *f* of the "infinitesimal interval" $\Delta + a$ obtained by translating Δ to *a* is straight and coincides with the tangent to the curve at $x = a$. Thus each real function has the effect of "bringing"



Δ into coincidence" with the tangent vector to the curve associated with the function at any point on it. In this sense, then, Δ plays the role of a *generic tangent vector.* Also, since the image of Δ under a function is necessarily a straight line and a part of the associated curve, it follows that each point on a curve is contained in a nondegenerate infinitesimal straight segment of the curve. In other words, in **SIA** *curves are infinitesimally straight.* This is the **Principle of Infinitesimal Straightness.**

In **SIA** there is a sense in which *everything is generated by the domain of infinitesimals.* For consider the set Δ^* of all maps $\Delta \to \Delta$. It follows from the Affineness Principle that **R** can be identified as the subset of Δ^* consisting of all maps vanishing at 0. In this sense **R** is "generated" by Δ. ExplicitlyΔ^* is a monoid under composition which may be regarded as acting on Δ by composition: for $f \in \Delta^*$, $f \cdot \varepsilon = f(\varepsilon)$. The subset *V* consisting of all maps vanishing at 0 is a submonoid naturally identified as the set of *ratios of infinitesimals.* The identification of **R** and *V* made possible by the principle of infinitesimal affineness thus leads to the characterization of **R** itself as the set of ratios of infinitesimals. This was essentially the view of Euler, who regarded infinitesimals as formal zeros and real numbers as representing the possible values of 0/0. For this reason **R** in **SIA** might be called the space of *Euler reals.*

Once one has **R**, Euclidean spaces of all dimensions may be obtained as powers of **R**, and arbitrary Riemannian manifolds may be obtained by patching together subspaces of these. This opens up the possibility of doing differential geometry in **SIA.**

Notice that the postulates of **SIA** are *incompatible with the Law of Excluded Middle of classical logic* (LEM) . The failure of LEM follows from the Cancellation Principle. To begin with, if $x \neq 0$, then $x^2 \neq 0$, so that, if $x^2 = 0$, then necessarily not $x \neq 0$. This means that

$$\textit{for all infinitesimal } \varepsilon, \textit{ not } \varepsilon \neq 0. \qquad (*)$$

Now suppose that LEM were to hold. Then we would have, for any ε, either $\varepsilon = 0$ or $\varepsilon \neq 0$. But (*) allows us to eliminate the second alternative, and we infer that, for all ε, $\varepsilon = 0$. This may be written

$$\textit{for all } \varepsilon, \ \ \varepsilon.1 = \varepsilon.0,$$

from which one derives by the Cancellation Principle the falsehood $1 = 0$. So again LEM must fail.

The "internal" logic of **SIA** is accordingly not full classical logic. It is, instead, *intuitionistic* logic. In practice when working in **SIA** one does not notice this "change of logic" because, like much of elementary mathematics, the topics traeted there are naturally thandledby constructive means such as direct computation.

To illustrate this point, I will derive in **SIA** a basic law of the differential calculus, the *product rule:*

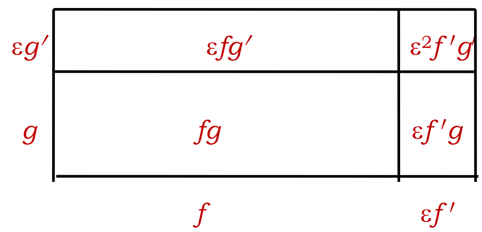
$$(fg)' = f'g + fg'.$$

To do this one computes

$$(fg)(x + \varepsilon) = (fg)(x) + (fg)'(x) = f(x)g(x) + (fg)'(x),$$

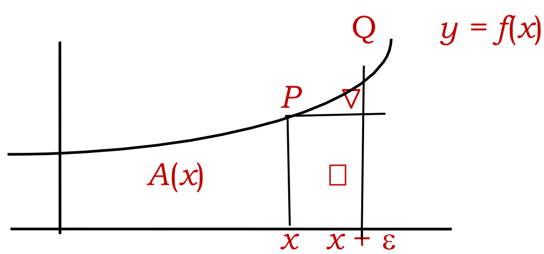
$$\begin{aligned}(fg)(x + \varepsilon) = f(x + \varepsilon)g(x + \varepsilon) &= [f(x) + f'(x)].[g(x) + g'(x)] \\ &= f(x)g(x) + \varepsilon(f'g + fg') + \varepsilon^2 f'g' \\ &= f(x)g(x) + \varepsilon(f'g + fg'),\end{aligned}$$

since $\varepsilon^2 = 0$. Therefore $\varepsilon(fg)' = \varepsilon(f'g + fg')$, and the result now follows from the Cancellation Principle. This calculation is depicted in the diagram below.



Infinitesimals in **SIA** have two fundamental aspects, *algebraic* and *geometric.* Algebraically, they are real numbers whose squares vanish; geometrically, they are straight segments of curves. Both of these aspects are used in applications.

The interplay of these aspects is well illustrated by the derivation in **SIA** of the *fundamental theorem of the calculus.* To this end, let **I** be a closed interval $\{x: a \leq x \leq b\}$ in **R** — or **R** itself — and let f: **I** → **R**; also let $A(x)$ be the area under the curve $y = f(x)$ as indicated in the figure:



Then

$$\varepsilon A'(x) = A(x + \varepsilon) - A(x) = \square + \triangledown = \varepsilon f(x) + \triangledown.$$

Now, by the Principle of Infinitesimal Straightness, the arc PQ is a straight line; accordingly $\triangledown$ is a triangle of area $\tfrac{1}{2}\,\varepsilon.\varepsilon\, f'(x) = 0$. It follows that $\varepsilon A'(x) = \varepsilon f(x)$, and the Cancellation Principle gives

$$A'(x) = f(x).$$

Thus, if one regards $A(x)$ as the integral of $f(x)$, the above equation asserts that differentiation is the inverse of integration – the fundamental theorem of the calculus.

This derivation, which is typical of such arguments in **SIA,** displays the following pattern. Suppose we are investigating the behavior of some variable quantity $F(x)$ (in the above derivation F is A). The approach taken in **SIA**, as in the differential calculus, is to begin the investigation by confining it initially to the infinitesimal world. Life in the infinitesimal world is beautifully simple: curves are just straight lines, and the squares of incremental changes vanish. This makes the determination of infinitesimal increments equally simple, enabling the increment $\varepsilon F'(x)$ in $F(x)$ to be presented in the form $\varepsilon k(x)$, where $k(x)$ is some explicit function whose form has been obtained by "infinitesimal" analysis. Thus we obtain an "infinitesimal" equation of the form $\varepsilon F'(x) = \varepsilon k(x)$. Applying the Cancellation Principle in turn yields the "differential" equation

(*) $$F'(x) = k(x)$$

which holds in the world "in the large".

The Cancellation Principle thus provides a formal, astonishingly simple link between the infinitesimal world and the world "in the large". The idea of a linkage between the two worlds was the animating principle behind applications of the calculus throughout the 17th and 18th centuries.

In practice, of course, the equation (*), while of fundamental importance, is only the first step in determining the explicit form of the function F. For this, it is necessary to "integrate" k, that is, to provide k with an *antiderivative,* or *integral,* an explicit function G such that $G' = k$. It will then follow that $F' = G'$, from which we will be able to conclude that $F = G$. Strictly speaking, F and G may differ by a constant function but for simplicity I'll ignore this.

To carry out this procedure in **SIA** one needs to introduce an additional postulate. I shall define a *stationary point* of a function $f\colon \mathbf{I} \to \mathbf{R}$ to be a point $a \in \mathbf{I}$ in whose vicinity "infinitesimal changes in the value of the argument fail to change the value of f", that is, for which $f(a + \varepsilon) = f(a)$ for all ε. This means that $f(a) + \varepsilon f'(a) = f(a)$, so that $\varepsilon f'(a) = 0$ for all ε, from which it follows by the Cancellation Principle that $f'(a) = 0$. Thus a stationary point of a function is precisely a point at which the derivative of the function vanishes.

In classical analysis, if the derivative of a function is identically zero, the function is constant. This fact is the source of the following postulate concerning stationary points adopted in **SIA**:

Constancy Principle. If every point in an interval J is a stationary point of $f: J \to \mathbf{R}$ (that is, if f' is identically 0), then f is constant.

It follows from the Constancy Principle that two functions with identical derivatives differ by at most a constant.

In ordinary analysis the continuum **R** is connected in the sense that it cannot be split into two non-empty subsets neither of which contains a limit point of the other. In **SIA, R** has the vastly stronger property of *indecomposability* or *cohesiveness:* it cannot be split *in any way whatsoever* into two disjoint nonempty subsets, Brouwer's term for this was "unzerlegbar". For suppose $\mathbf{R} = U \cup V$ with $U \cap V = \varnothing$. Define $f: \mathbf{R} \to \{0, 1\}$ by $f(x) = 1$ if $x \in U$, $f(x) = 0$ if $x \in V$. We claim that f is constant. For we have

$$(f(x) = 0 \text{ or } f(x) = 1) \quad \& \quad (f(x + \varepsilon) = 0 \text{ or } f(x + \varepsilon) = 1).$$

This gives 4 possibilities:

(i) $f(x) = 0 \ \ \& \ \ f(x + \varepsilon) = 0$

(ii) $f(x) = 0 \ \ \& \ \ f(x + \varepsilon) = 1$

(iii) $f(x) = 1 \ \ \& \ \ f(x + \varepsilon) = 0$

(iv) $f(x) = 1 \ \ \& \ \ f(x + \varepsilon) = 1$

Possibilities (ii) and (iii) may be ruled out because f is continuous. This leaves (i) and (iv), in either of which $f(x) = f(x + \varepsilon)$. Thus any point x is a stationary point of f and it follows from the Constancy Principle that f is constantly 1 or 0. In the first case $V = \varnothing$, and in the second $U = \varnothing$.

Put succinctly, the Constancy Principle asserts that "universal infinitesimal (or "local") constancy implies global constancy", or "infinitesimal behaviour determines global behaviour" The Constancy Principle brings into sharp focus the difference in **SIA** between points and infinitesimals. For if in the Constancy Principle one replaces "infinitesimal constancy" by "constancy at a point" the resulting "Principle" is false because *any function whatsoever* is constant at every point. But since in **SIA** all functions on **R** are smooth, the Constancy Principle embodies the idea that for such functions local constancy is sufficient for global constancy, that a nonconstant smooth function must be somewhere nonconstant over arbitrarily small intervals.

The Constancy Principle provides another bridge between the infinitesimal world and the world "in the large", the lack of which, as you have claimed, doomed the idea of infinitesimal, leading to its inevitable replacement by the limit concept.

In **SIA** the Constancy Principle reconnects the infinitesimal and the extended. Behaviour "in the large" is completely determined by behaviour 'in the infinitely small'. "

I wound up my impromptu lecture, which I was pleased to see Weyl had been following with rapt attention, by asking him what he thought of the theory I had presented.

"That was most interesting indeed," he responded. "Smooth infinitesimal analysis as you have outlined appears to be an elegant approach to infinitesimal analysis. The key notion is that every function 'in the small' is affine, so that, in particular every curve 'in the small' is a straight line. And I agree that the principles smooth infinitesimal analysis as you have presented it ds embody a link between the infinitely small – at least when that is construed as having a vanishing square - and the world 'in the large'. Of course, the world 'in the large' in smooth infinitesimal analysis is essentially that of classical physics and differential geometry. It is very striking that this world, in which every function is smooth, can in some sense be 'reduced' to its infinitesimal parts. Also striking is that logic in this world is intuitionistic. Leibniz would have been shocked to learn that his principle *natura non facit saltus,* which is realized in smooth infinitesimal analysis, is incompatible with the law of excluded middle! Very congenial to me also is the indecomposability of the continuum. But is smooth infinitesimal analysis consistent? And did you invent it yourself?"

"It is hardly my invention!," I replied," I learned about it from certain mathematicians of my acquaintance who are still in the process of developing it. They have succeeded in showing that it is consistent if classical analysis is."

At this point I could see from the puzzled expression on Weyl's face that he was beginning to wonder just who I was and where I had acquired my mathematical knowledge. I realized with dismay that I had got carried away and overstepped the mark. I quickly rose to bring our colloquy to an end. "Professor Weyl, it's been an honour to meet you and I have greatly enjoyed our discussion. You have also been most gracious in listening to my impromptu lecture."

Weyl also rose and said, with a gleam in his eye, said "I too enjoyed our discussion, it has been most stimulating. Your little lecture was unusual, to say the least. It has led me to suspect -please take no offense- that it could not have been delivered by an ordinary member of the staff of the *Scientific American,* excellent as that publication is. But no matter - I intend to give some thought to smooth infinitesimal analysis!"

We shook hands, bade each other farewell, and I departed.

F. William Lawvere

My investigations into 20th century mathematics had shown that the emergence of the theory of categories was a pivotal development. After its invention by Eilenberg and Mac Lane in 1945, category theory began to evolve into a unifying scheme for the whole of mathematics, and even came to rival set theory as a foundation for mathematics.. Playing a major role in this evolution was the American mathematician Francis William Lawvere. Like Hermann Weyl, Lawvere was one of the few mathematicians in whose thinking philosophy played a role of genuine importance. But from their writings, even though chiefly confined to mathematics, it was clear that the two were poles apart in their underlying philosophical convictions. They also saw the relationship between mathematics and philosophy quite differently. Their differing views in this regard led the one to a reluctant pessimism, the other to an unwavering optimism. Weyl's philosophical sensibilities played an inhibiting role in his mathematical activity, remarkable though that was. He required mathematics to have the same degree of phenomenological certainty as the intuitive self-evidence provided by the subjective ego, the Cartesian *cogito.* Mathematics outside what he called the "lighted circle of intuition" lacked, for him, the intuitive certainty he craved. He took the paradoxes of set theory seriously as an indication that mathematics had overstepped its intrinsic philosophical boundaries, and confined his mathematical activity mainly to those areas of mathematics which in his view, remained within those boundaries. But he did not have the same attitude to (mathematical) physics, to which he made many important contributions. He believed that the physical world outside immediate subjectivity could not be grasped directly, but could only be represented by means of symbols, through a process he called "symbolic construction". His abiding conviction of the harmony of the inner and outer worlds led him to believe that this process of symbolic construction would reflect that harmony and so the resulting mathematical physics would not be threatened by the contradictions generated in pure mathematics by the unfettered use of the imagination.

Weyl's uncertainties concerning mathematics had a natural origin in his conviction in the primacy of the subjective. He thought that the contradictions arising in set theory and logic ultimately resulted from an illegitimate extension of the immediate givenness of the subjective to the speculative wonders of the transcendent. He wrote, for example,

We are less certain than ever about the ultimate foundations of (logic and) mathematics; like everybody and everything in the world today, we have our 'crisis'. We have had it for nearly fifty *years. Outwardly it does not seem to hamper our daily work, and yet I for one confess that it has had a considerable practical influence on my mathematical life: it directed my interests to fields I considered relatively 'safe', and it has been a constant drain on my enthusiasm and determination with which I pursued my research work. The experience is probably shared by other mathematicians who are not indifferent to what their scientific endeavours mean in the contexts of man's whole caring and knowing, suffering and creative existence in the world.*

And

The ultimate foundations and the ultimate meaning of mathematics remain an open problem; we do not know in what direction it will find its solution, nor even whether a final objective answer can be expected at all. 'Mathematizing' may well be a creative activity of man, like music, the products of which not only in form but also in substance defy complete objective rationalization.

Lawvere does not mention Weyl in any of his writings that I could discover, but it seemed clear to me that, whatever his opinion of Weyl as a mathematician, he would have rejected emphatically the sentiments Weyl expresses in these quotations. By contrast, Lawvere's writings, even at their most esoteric, never failed to reflect his robustly positive attitude towards mathematics and what he saw as its true purpose – shared with all genuine sciences - the unveiling of the structure of the objective world. This attitude was, I took it, reinforced by his early readings of Marx, Engels and Lenin, through which he had become a convinced dialectical materialist. His firm belief in this philosophy enabled him to avoid the doubts concerning the soundness of mathematics which plagued Weyl throughout his life. Along with the founders of dialectical materialism, Lawvere saw mathematics, not, as did Weyl, as a subjective source for "symbolic construction" of the objective world, but as a direct expression of the objective world as reflected in subjectivity.

At university Lawvere initially studied engineering physics. He began the serious study of mathematics with the aim of clarifying the foundations of continuum physics, a topic in which he had developed a particular interest. In the course of his mathematical studies he soon came to learn about the recently created theory of categories and functors, and became convinced that category theory was capable of providing a unifying scheme and a foundation for the whole of mathematics, both pure and applied. His powerful convictions in this regard quickly led, inter alia, to his categorical analysis of mathematical logic and theory, and later to his and Myles Tierney's formulation of the amazingly fertile concept of elementary topos.

As a dialectical materialist he began to forge links between dialectics and category theory, between mathematics and philosophy, in which ideas from the former were to furnish philosophical meanings for concepts in the latter, and, reciprocally, concepts in the latter precise mathematical formulations of ideas in the former. From his published writings, and reports of his lectures, it is clear that he had come to a state of absolute certainty, which he was never to lose about the truth of both sides of this linkage, as well as that of the linkage itself.

Lawvere regarded category theory as an advance as important in its way as the invention of the calculus, but at a vastly more general level reflecting the subsequent advances in mathematics. Another important purpose he assigned to mathematics, as a true science, was its use as a weapon in what he saw as the perpetual battle (or, as he would say, dialectical opposition) between knowledge and belief, between the objective and the subjective. He shaped category theory into a highly sophisticated weapon for use in this battle. In a commentary on an early paper of his, he writes:

The original aim of this article was to demystify the incompleteness theorem of Gödel and the truth-definition theory of Tarski by showing that both are consequences of some very simple algebra in the cartesian-closed setting [of category theory]. It was always hard for many to comprehend how Cantor's mathematical theorem [here Lawvere is referring to Cantor's famous theorem that a set always has more subsets than elements] could be re-christened as a"paradox" by Russell and how Gödel's theorem could be so often declared to be the most significant result of the 20th century. There was always the suspicion among scientists that such extra-mathematical publicity movements concealed an agenda for re-establishing belief as a substitute for science. Now, one hundred years after Gödel's birth, the organized attempts to harness his great mathematical work to such an agenda have become explicit.

Lawvere could equally have used the term "objectification" here instead of "demystification". For he is not denying the importance of Gödel's (or, in this case, Tarski's) mathematical work - although he would have disagreed totally with Godel's platonist, and ultimately religious, philosophical outlook. But ultimately Lawvere is claiming that Gödel's incompleteness theorems and Tarski's undefinability of truth theorem have only a subjective significance, and their objective mathematical content, while non-trivial, is comparatively unproblematic. It is the subjective aspect of these results which he saw as providing fodder for what he identifies as 'extra-mathematical publicity movements" bent on "re-establishing belief as a substitute for science". Lawvere's writings contain a number of warnings concerning these "movements". In particular he took aim at the controversial John Templeton Foundation, which he said “attempts to inject religion and pseudo-science into scientific practice” when it sponsored the international conference organized by the Kurt Gödel Society in honour of the celebration of Gödel’s 100th anniversary.

Although Lawvere did initially publish mathematical works in the traditional “theorem -proof” form, many of his later publications are remarkable fusions of pronunciamento and meditation, yet remaining (just) within the boundaries of mathematics. These writings, which read like compressed surveys of the history of mathematics, viewed through the lens of category theory, I found intriguing , especially when, as an ex-mathematician, I could not understand the derails underlying the mathematical insights reported with such authority. So it was natural that I became particularly interested in talking to the author himself. But it had become clear to me that there was a fundamental difference in our attitudes towards the nature of reality, I sharing with Weyl and Sartre the belief in the primacy of the subjective, inner world, Lawvere sharing with Marx, Engels and Lenin the belief in the primacy of the objective, outer world. I took it that none of us on either side would deny the actual existence of what the members of the other side claimed to be primary, only the primacy thereof. Having thought this through, I felt prepared for our encounter.

After some consideration, I thought that an appropriate venue for our potential discussion would be the International Conference on the Ramifications of Category Theory held in Florence, Italy in November 2003. Lawvere gave a series of invited lectures at this conference. Its numerous participants included both mathematicians and philosophers. Among these were the logicians Dana Scott and Angus Macintyre, and the mathematician Pierre Cartier.

I figured that I would be able to blend in with the participants and attend the various lectures without attracting attention. I planned to approach Lawvere at the end of the proceedings, present myself as a member of the editorial board of the mathematical journal the Mathematical Intelligencer, and propose an interview. Accordingly I time-travelled to Florence a day before the conference was due to start and booked a room for ten days in a modest pensione near the central railway station.

The meeting, which was organized by Alberto Peruzzi, a philosopher at the University of Florence, was lively, and the participants very approachable and friendly. As I had hoped, my presence there remained unquestioned. I attended the various lectures given at the conference, including, of course, those of Lawvere, which were on the whole illuminating but some parts of which I confess I found difficult to understand.

I was able to attend the conference dinner, which was held at an excellent restaurant in central Florence. Good cheer reigned. I found myself seated next to John L. Bell, an Anglo-Canadian logician who had given one of the less technical talks at the meeting, on differing views of the continuum. He turned out to be a highly voluble fellow and we fell quickly into animated conversation, from which it emerged that we had many common interests - mathematics, music, philosophy, painting, film among others. The uncovering of these common interests was largely the result of my interlocutor's continually changing the topic of conversation through what seemed a process of free association. At some point I volunteered the information that I was a functionary from the *Mathematical Intelligencer* bent on obtaining an interview with Lawvere. I was struck by Bell's response: that he was glad that the *Intelligencer* was going to devote some attention to Bill Lawvere, an unorthodox and visionary mathematician, a true American original. As the *dolce* was served, glasses were tapped, calling the gathering to order. A man whom I recognized as Dana Scott, the illustrious American logician, rose and proceeded to make some observations. After thanking the organizers of the meeting and saying how stimulating and enjoyable it had been, he remarked that for him the only discordant note in the proceedings had arisen at the hotel in which he and some other participants had been billeted. Every night, he said ruefully, the quiet had been shattered by the sound of a group of rock musicians engaging in a jam session. He went on to suggest that a slick name for the group, given the title of the conference , would be "The Ramcats". This brought the house down.

On the last day of the conference, after the final session, I approached Lawvere and told him my prepared story. I suggested that, if he was willing, we could meet for dinner in a restaurant of his choosing where the interview could take place. I was delighted when he readily agreed and proposed that we meet the next day at a nearby restaurant.

At the agreed time we both sat down at a table in the restaurant, and I initiated our discussion.

"Professor Lawvere, thank you for agreeing to talk with me. Through your visionary mathematical work in category theory you have acquired the reputation among mathematicians as being a virtual embodiment of that subject. However, it is true to say, is it not, that you do not regard your mathematical speciality as a thing in itself to be studied merely for its own sake, but rather as a philosophical tool for understanding the nature and structure of the world, indeed of the whole of Being. In this respect your goal as a mathematician is, it seems to me, closer to that of a philosopher. Would you agree?"

"Yes," Lawvere replied, "but that is largely because over historical time a division of labor has arisen between mathematics and philosophy. In Ancient Greece the Pythagoreans introduced the term 'mathematics' from a root which means learning, study, science, or, by extension, knowledge. All these terms apply equally well to philosophy. If philosophy is the science of Being in all its aspects, then mathematics is the science of certain fundamental aspects of Being, namely, quantity, space, quality, change and thinking. Unfortunately, the scope of philosophy has expanded, and so diluted to the point that it has become more a source of distraction than of wisdom. The more disciplined nature of mathematics has prevented it from suffering a similar fate, but even so philosophers have attempted to distort the science of mathematics for their own purposes."

"I note," said I, "that you include thinking as one of five aspects of Being falling under mathematics. While the first four of these aspects (with the possible exception of quality) are objective, or mind-independent at least, thinking diverges radically from these in requiring a thinking subject. Now you will recall that in 1854 Boole published his famous work *An Investigation of the Laws of Thought, on which ate Founded the Mathematical Theories of Logic and Probabilities.* Would you say that your reasons for regarding thinking as describable in mathematical terms are similar to Boole's?"

"Well , at least to some extent, " Lawvere responded, "Boole set himself the admirable goal of establishing the science of logic on a mathematical foundation. He understood laws of thought in the traditional sense as being those governing valid thought or valid inference, that is, logic. He was not concerned with the actual process of thinking or of the thinking subject as such. Now I am convinced that the thinking process is a constitutive element of Being, along with the other elements I mentioned. But thinking has a special property lacked by those others, namely its capacity to reflect Being, that is, the ability to assimilate certain aspects of Being external to thinking and thereby guide the actions of the thinking subject. Both the Subjective and the Objective are concepts within Thinking, reflecting those aspects of Being which are dependent or independent, respectively, of any thinking subject. These in turn give rise to what Hegel termed subjective and objective logic. Objective logic is a codification of the constitutive principles underlying the constructions of the concepts and their interactions which grow out of the study of space and quantity, or, in general, any serious object of study."

Lawvere paused to light a cigarette and continued, "Subjective logic, on the other hand, is just the logic of inference between statements, essentially the logic studied by Boole. Here statements are of interest only for their potential in delineating the objects which embody the concepts treated by objective logic. Since this is clearly a serious object of study, it follows that subjective logic can be regarded as a part of objective logic. Subjective logic reflects and helps to guide construction in objective logic. In fact, this relationship is precisely the form taken by the reflection within Thinking of Thinking itself, considered as part of Being. In short, Being is to Thinking as objective logic is to subjective logic."

"Nicely put," I said. "It's interesting that, in the realm of logic, you consider the subjective as being a part of the objective. This reminds me of Husserl's - a mathematician like you, in origin at least - contention that his program of phenomenoloy is a science, the science of objective subjectivity, to be included among the genuine sciences. Like you, Husserl maintained that subjectivity and consciousness, and thinking generally, have an objective content. However, the method he proposed for investigating that content, differed from the experimental-deductive method employed by natural scientists, although it was intended to be no less objective. Husserl envisaged a science of subjectivity - phenomenology - that is, so to speak, absolutely subjective, in analogy with the traditional view of natural scientists that the material world is absolutely objective. Just as the natural scientist investigates the material world, treated independently of the subjectivity of the investigator, so, inversely, the intent of Husserl's phenomenologist is to investigate the subjectivity of the investigator, treated independently of the existence of the material world. Husserl saw that this symmetry could not be regarded as complete unless subjectivity was furnished with a correlate as objective as the material world is ordinarily taken to be. So he was, reluctantly perhaps, led to embody the subjective in the form of an 'ego of pure consciousness', the investigation of whose structure, taken as objective, is the desired correlate to the natural scientist's probing of the material world. But he was uncomfortably aware that in making this analogy between the ego of pure consciousness and the external world of matter the philosopher ran the risk of having to attribute a uniqueness, a singularity to the ego similar to the uniqueness which is normally ascribed to the physical universe. But the uniqueness of the ego could mean just one of two things, either that there exists only one individual ego, namely, mine, or there exists just one universal ego. The first alternataive is the doctrine of solipsism, and the second the doctrine of universal consciousness, a tenet of the ancient Indian philosophy of Vedantism. Husserl found neither of these alternatives acceptable, and he never really succeeded in resolving the issue."

At this point Lawvere interjected, "I refuse to wallow in subjectivity! I am convinced that subjectivity and consciousness, while unquestionably real, cannot exist independently of the material world. Our own experience as conscious beings continually confirms this."

"So you'd endorse Dr Johnson's stone-kicking refutation of Berkeley's subjective idealism."

"Absolutely! It's a perfect example of scientific experimentation confirming to the subject the objective existence of the material world. But let us return to Husserl. I can't claim to have a real knowledge of phenomenology although I have heard of it. From its name I had thought it to be just another form of subjective idealism, and so I ignored it. But your description of it leads me to think that I may have been mistaken. From what you say it seems that Husserl was indeed concerned to render the subjective objective, and treat its objective investigation as a science. However, it seems that Husserl's was based on a detaching of the mind from the rest of Being, and so was bound to collapse into solipsism or expand into the objective idealism of a universal consciojusness."

"It would seem," I said, "that I ascribe more importance to subjectivity than you do. But that need not detain us. Now, your philosophical outlook can be described as dialectical. Let me ask you how you would characterize dialectics."

"Well, " Lawvere returned , "Perhaps the most basic sense of 'dialectics' is this: in order to understand a situation which unites two opposing aspects, the essential thing is start by recognizing each aspect and the relationship between them, rather than to set out from the beginning to prove that one aspect is everything and the other one is nothing. This idea underlies Hegel's assertion that the purpose of dialectics is 'to study things in their own being and movement'. For Hegel the essence of the dialectical method lay in the objective movement of concepts. This movement takes place in objective time but its driving force is the overcoming of opposites or contradictions. As Hegel puts it, the result of the dialectical process is a new concept, but one higher and richer than the one preceding it—richer because it negates or opposes its predecessor and yet contains it, as the unity of itself and its opposite. This is Hegel's doctrine of *Aufhebung*. For Hegel, 'the dialectical constitutes the moving soul of scientific progress'. Later Marx and Engels, who regarded Hegel's formulation of the dialectical method as being too idealistic, modified it by replacing the movement of concepts by the movement of the material world. While for Hegel the real world is only the external, phenomenal form of 'the Idea', Marx and Engels maintained that on the contrary, the ideal is just the material world reflected by the human mind, and translated into forms of thought. This led to the philosophy of dialectical materialism. Engels proclaimed that the world is not to be comprehended as a complex of ready-made things, but as a complex of processes, in which the things, apparently stable no less than their mental images in our heads, the concepts, go through an uninterrupted change of coming into being and passing away. Dialectical philosophy is nothing more than the reflection of this process in the thinking brain. Dialectics is, as Marx put it, the science of the general laws of motion, both of the external world and of human thought. Lenin identified the essence of dialectics as the splitting in two of a single whole and the grasping of its contradictory parts."

"In *Materialism and Empitiocriticism*," I observed, "it seems to me that Lenin is more concerned with the materialist aspect of dialectical materialism than with the dialectical aspect."

"Yes, you are right," Lawvere responded. "And there was good reason for that, since Lenin's fundamental concern at the time it was written was to combat the philosophical idealism, with its anti-scientific overtones, which he saw was affecting some of his Bolshevik comrades. Thus he decisively characterized the world as being composed of matter-that-moves and matter-that-thinks, with the latter playing the particular role of reflecting the decisive relations in the world in order to provide theory as a guide to action. This materialist world-picture is in direct opposition to the anti-scientific world-pictures of idealism, both objective and subjective. Subjective idealism was concocted by Berkeley in order to prepare the way for the acceptance of objective idealism, and this destructive and anti-scientific work has been carried on in the 20th century by Mach, Russell, Brouwer, Heisenberg, among others. The science of mathematics has subjected to particular distortions by these idealists as part of their efforts to get the public to accept their philosophy that the world is a figment of the imagination - whether ours or 'God's'. "

“Evidently you share Lenin’s animus towards idealism,” I said. “But let us return o dialectics. In *Dialectics of Nature* Engels formulates the three laws of the dialectic: the law of the unity of opposites, the law of the transformation of quantity into quality, and the law of the negation of the negation. Do you regard these as useful?”

“Engels distilled his three laws from his reading of Hegel's *Science of Logic*. They are useful slogans for communicating the essence of dialectics to the people, something that Hegel, as an ivory tower academic, had no interest in doing. Of course the truly fundamental law, which in essence goes back to Heraclitus, is the law of the unity of opposites. The second law is immediately derivable from it, since quantity and quality are themselves opposites. The law of the negation of the negation is a stripped-down version of Hegel’s *Aufhebung*.”

“What was the source of your interest in dialectical philosophy, and what was its impact on your understanding of mathematics?”

“In my youth my chief interests were in engineering and physics, but at the same time I was intrigued by the speculative ideas I found in science fiction. The precision and clarity of mathematics also appealed to me. I first went to university to study engineering physics, and I enjoyed the experimental courses offered, but the theoretical courses struck me as lacking in the precision I found in mathematics. So I resolved to study mathematics, not really for its own sake, but as an instrument for deepening my understanding of the world. However, mathematics did not fully quench my youthful thirst for understanding. I became interested in history and how historical forces shape the social world in which we are immersed, for better or worse, in all its contradictory aspects.”

“Did the Vietnam war and the political turbulence of the 1960s have an effect on your thinking?”

“Of course, the Zeitgeist of the 1960s, pervaded by the Vietnam war was, as for many of my contemporaries, was an important catalyst. I also admired, if only from afar, the success of the Chinese communist revolutionaries, led by Mao Tse Tung, in overthrowing the imperialist yoke on their country. But fundamentally it was for philosophical reasons that I was led to study the dialectical and historical materialism of Marx, Engels and Lenin. It was through my reading of Lenin’s *Conspectus of Hegel’s Science of Logic* that I first encountered Hegel’s dialectical philosophy. These studies confirmed and enhanced my understanding of mathematics - that it is, first and foremost, the science of space forms and quantitative relationships, and its fundamental purpose is to clarify this relationship in order to act as a basis of unity of people in solving problems (not mathematical problems) in the struggle for useful production, and in the conscientiousness associated with this struggle, which is scientific experimentation. I saw also that mathematics has an underlying logic, whose purpose is to clarify and simplify its learning, use and development. But I also came to see that, in a dialectical way, there is also a counterpurpose, namely to obscure, complicate and prevent the learning, use and development of mathematics, in particular, to freeze the development of mathematics by forcing everything into a preconceived static framework. Both of these purposes are fighting with each other inside each of us. Often the counterpurpose wins over the purpose. This happens because the counterpurpose is in the interests of the present ruling class, the monopoly capitalists, who oppose the development of useful production."

"It would be fair, then, to characterize your philosophical outlook, and your views on the nature of mathematics, as fundamentally Marxist-Leninist.

"Yes,that's right. I have long accepted the Marxist-Leninist account of the nature of reality - dialectical materialism- as fundamentally correct."

"Very good," I said . "Now thus far, philosophy has been the principal focus of our discussion. But it is mathematics, and above all category theory, that you have made your life's work. You did your doctoral work in category theory at Columbia with Samuel Eilenberg, one of the founders of category theory. How did you first come to learn about category theory, and what was its appeal for you?"

"My resolve to study mathematics as an undergraduate at Indiana University was encouraged by Clifford Truesdell, who was both a mathematician and an expert on continuum physics, which had come to interest me greatly. He emphasized the fundamental importance of mathematics in physics. It was from him that I first learned of Galileo's famed observation that the book of nature is written in the language of mathematics. So I decided to study mathematics. I learned a lot from my reading of John L. Kelley's book on General Topology, which covered many important mathematical topics. Toward the end of the book Kelley refers to the 'galactic' method' in connection with the study of rings of continuous functions, mentioning its successful use by Eilenberg and Steenrod in their axiomatic treatment of homology theory. At first I thought this approach too abstract but a fellow student convinced me of its importance.

This 'galactic' method was, in fact, the Eilenberg-Mac Lane theory of categories and functors! My fellow student and I we discussed their potential for unifying and clarifying mathematics of all kinds. This was my first introduction to category theory. I soon came to see that category theory would be of real use in simplifying the foundations of continuum physics. In Summer 1958 I studied Topological Dynamics with George Whaples, with the intention of understanding as much as possible in categorical terms. When Truesdell asked me to lecture for several weeks in his 1958-1959 Functional Analysis course, it quickly became apparent that very effective explanations of such topics as Rings of Continuous Functions and the Fourier transform in Abstract Harmonic Analysis could be achieved by making explicit their functoriality and naturality in a precise Eilenberg-Mac Lane sense. I developed categorical thinking on my own and found that some of my ideas had already been much more fully worked out Kan under the name of 'adjoint functors', and by Grothendieck as 'fibered categories'. Thus category theory became the central focus of my study. But I was bothered by the frequent references in the categorical literature to some mysterious difficulty in basing category theory on the traditional set theory. I had gained some initial understanding of mathematical logic through my following a course on Kleene's book 'Introduction to Metamathematics' and my many discussions with Max Zorn, whose office was adjacent to mine. I concluded that the solution to the foundational problem in category theory would be to develop an axiomatic theory of the Category of Categories. I took the first steps in this direction in my Ph.D. thesis at Columbia in 1963. After completing my thesis I continued to work on axiomatizing the Category of Categories, which led to my 1966 paper on the topic."

I said, "In your thesis you introduced a number of other categorical ideas which have become well known."

"One of my principal aims, "Lawvere responded, "was to provide the theory of models with a categorical formulation. In my thesis I did this by introducing what I called algebraic theories, the categorical counterparts of equational theories. Here the key insight was to view the logical operation of substitution in equational theories as composition of arrows in a certain sort of category. I showed how models of such theories can be naturally identified as functors of a certain kind, so launching the development of what I called functorial semantics."

"After completing your thesis, you developed what you called the elementary theory of the category of sets (ETCS). Could you describe how this came about?"

“Well, soon after I started to learn category theory and began to consider the possibility of developing the category of categories as foundation for mathematics, it occurred to me that set theory, which most mathematicians accepted without question as the foundation for their professional activity, should itself be provided with a categorical formulation. Since sets and mappings themselves form a category, I was led to develop a system of axioms characterizing that category. ETCS was this system of first-order axioms. The theory characterizes the category of sets and mappings as an abstract category in the sense that any model for the axioms that satisfies the additional non-elementary axiom of completeness, in the usual sense of category theory, can be proved to be equivalent to the category of sets. More importantly, the theory provides a foundation for mathematics that is quite different from the usual set theories in the sense that much of number theory, elementary analysis, and algebra can be developed within it even though no relation with the usual properties of membership can be defined.”

“Didn’t your idea become known among mathematicians as ‘sets without elements’?”

“I never proposed the idea of ‘sets without elements!’” Lawvere rejoined. “But the slogan caused many misunderstandings over the next 40 years because Saunders Mac Lane got a kick out of the phrase and enjoyed repeating it. When I produced a streamlined form of the axioms Mac Lane communicated it to the Proceedings of the U.S. National Academy of Sciences. I never denied that sets in the intuitive sense have elements - except of course for the empty set! What I argued was that, from a mathematical standpoint the conception of a set should not be tied to the membership or element hood relation, where a set is substantially identified as just the ‘collection’ of its elements. This conception of set has led to the mathematically irrelevant cumulative hierarchy of sets of sets, sets of sets of sets,… ad infinitum. I came to the view that the set concept would serve the needs of mathematics better if it were to be defined using as a primitive the idea of map and map composition, that is, in categorical terms. This categorical formulation, in which the idea of a universal mapping property as an adjunction is central, gives concrete expression to the persistent feeling among algebraists and topologists that the essence of mathematics at all levels lies not in Substance but in Form, that is, in isomorphism-invariant structure.”

“As a matter of fact,” Lawvere continued, “ECTS arose in the first place as the result of a purely practical educational need. When I began teaching at Reed College in 1963, I was instructed that first-year analysis should emphasize foundations, with the usual formulas and applications of calculus delayed until in the second year. Since part of the difficulty in learning calculus stems from the rigid refusal of most textbooks to supply clear and explicit statements of concepts and principles, I was very happy with the opportunity to oppose that unfortunate trend. I devoted part of the summer of 1963 to designing a course based on the axiomatics of Zermelo-Fraenkel set theory (even though I had already concluded that the category of categories is the best setting for 'advanced' mathematics). I spent several preparatory weeks trying to devise a calculus course based on Zermelo-Fraenkel set theory. But I soon realized that not even an entire semester would be sufficient for explaining all the (for a beginner bizarre) membership-theoretic definitions and results, then translating them into operations usable in algebra and analysis, then using that framework to construct a basis for the material on metric spaces I planned to present in the second semester. But I found a way out of the ZF impasse, making it possible for the able Reed students to take advantage of the second semester course I had planned. That was to present in a couple of months an explicit axiomatic theory of the mathematical operations and concepts (composition, functionals, etc.) as actually needed in the development of the mathematics. This crystallized into ECTS. Later I came to see that ETCS is essentially a categorical formulation of Cantor's concept of 'cardinal number'."

"Could you enlarge on this?"

"Sure. Actually it was John Myhill who suggested to me the analogy between ECTS and Cantor's 'cardinal numbers. I had been giving an introductory course at Buffalo on set theory based on ETCS, in which I referred to the objects of the category of sets as *abstract* sets. I emphasized that an abstract set can be conceived of as a bag of dots which are devoid of properties apart from mutual distinctness. Further, the bag as a whole was assumed to have no properties except cardinality, which amounts to the assertion that it might or might not be isomorphic to another bag. After hearing this description, John Myhill, who attended the first few lectures, said to me that he had seen all this before in Cantor. Later he brought along his copy of Cantor's works with a note referring me to Cantor's use of the term *lauter einsen,* which means, roughly, 'pure ones', in connection with his concept of *Kardinalen,* or 'cardinal numbers'. Myhill had noticed that Cantor's description of *Kardinalen* as *lauter einsen* and my description of abstract sets as 'bags of dots' were essentially the same."

"That's a nice example of what Leibniz would call preestablished harmony," I said. "I'd like now to discuss two of your most fruitful conceptions, synthetic differential geometry and elementary toposes. Could you describe how these ideas evolved and explain the connection between them?"

“Sure,” Lawvere returned. “I had studied Grothendieck’s theory of Abelian categories and felt that it should have a ‘non-linear’ analogue whose examples would include categories of sheaves of sets….”

“Sorry to interrupt, but what do you mean by ‘non-linear’?”

“The axioms of an Abelian category were designed to capture the abstract features of the category of Abelian groups, and more generally, of the concrete categories arising in linear algebra such as those of vector spaces and modules over a ring. There is a theorem, due to Barry Mitchell and Peter Freyd, which asserts that any Abelian category is ‘linear’ in the sense that it can be considered as a subcategory of a category of modules. By ‘non-linear’analogue I meant an axiomatic description of a category which captured the abstract features of categories of sheaves. Sometime in 1964 I wrote down some of the properties that I thought such categories should have. It was only in the summer of 1965 that I learned from Verdier that he, Grothendieck and Giraud had developed a full-blown theory of such non-linear categories under the term ‘toposes’ or ‘topoi’. However, this theory lacked the autonomy I sought.”

“Forgive me for interrupting again, but what do you mean by ‘autonomy’ here?”

“Well, a number of Giraud’s topos axioms were “higher-order” in that they made explicit reference to arbitrary sets of objects or maps. This made their meaning dependent on the properties of the universe of sets. By ‘autonomy’ I meant an independence of the notion of “arbitrary set”, in other words, a theory given a first-order or elementary formulation. Anyway, I later met Peter Gabriel and learned from him many aspects, not widely known even now, of the Grothendieck approach to geometry. I became convinced that topos theory, appropriately axiomatized in an elementary way could provide a unification of ideas coming from differential geometry, set theory and logic. I envisaged that a simple axiomatic theory of the topos concept would unify the apparently totally different concepts of infinitesimal motion in physics and differential geometry and the Boolean-valued models used in proving the independence of the continuum hypothesis. This only became a reality later on.”

“And what was the origin of synthetic differential geometry?”

"Synthetic differential geometry, or my conception of it, came about through my attempts to justify older intuitive methods in differential geometry. As an assistant professor in Chicago in 1967, Mac Lane and I taught a course on Mechanics, during which I began to think about this question. I also gave a separate advanced lecture series, attended by, among others, Mac Lane and my then student Anders Kock, in which I began to apply the Grothendieck topos theory I had learned from Gabriel to the problem of developing simplified foundations of continuum mechanics as I had initially been inspired by Truesdell's teachings. My particular contribution was to elevate certain ingredients, such as the representing object for the tangent bundle functor, to an axiomatic level so as to allow free development unencumbered by ad hoc constructions. It was immediately clear that the program would require the development, in a similar axiomatic spirit, of the topos concept. A few years later this idea bore fruit with Myles Tierney's and my axiomatic formulation of elementary toposes. But before this came about Anders Kock and I had already come to see that the essential features of differential geometry - what we called synthetic differential geometry - could be captured by axiomatizing, within a Cartesian closed category of 'spaces', the properties of the representing object for the tangent bundle functor. This object *D*, the generic tangent vector, is a part of the real line containing the origin 0 which is necessarily infinitesimal in a precise sense. It is a realization of the old idea of a tangent to a curve as an 'infinitesimal secant.' The key axiom for *D* that we formulated became known as the 'Kock-Lawvere axiom'. It asserts that every map from *D* to the real line is affine. This has two consequences: one qualitative, the other quantitative. Qualitatively, *D* is an infinitesimal space so small that remains 'straight' and 'unbroken' under any spatial transformation. Quantitatively, *D* 'consists' of the 'infinitesimal' real numbers whose squares are zero. Kock and I saw that the axiom we had arrived at led very quickly to the basic concepts and formulas of the differential calculus, based on a concept of infinitesimal which we felt was more natural than that within Abraham Robinson's nonstandard analysis. "

I said, "So synthetic differential geometry was essentially axiomatized before the axiomatization of elementary toposes."

"Yes, and for good reason," Lawvere responded. "As I've said, the novel idea of synthetic differential geometry, from a categorical point of view, is the representability of the tangent bundle functor by means of an 'infinitesimal' object *D*. And that, fundamentally, was the idea behind synthetic differential geometry. Developing the basics of SDG required only a Cartesian closed category of 'spaces' whose objects included the real line and its infinitesimal subobject D. These ideas were developed in 1967 before Tierney and I formulated the concept of elementary topos. Now the key idea underlying elementary toposes also involved representability of a functor, in this case the power set functor. This functor, as an operation at least, was important to set theorists and logicians, but not to geometers or topologists. When Grothendieck and his colleagues formulated the topos concept, they missed the fact that categories of sheaves and presheaves, of which the topos concept is a generalization, are actually closed under the forming of 'power sets' characterized in an appropriately categorical way. In 1968-69 I discovered the nature of the power set functor in toposes as a result of investigating the problem of expressing in elementary terms the operation of forming the associated sheaf from a presheaf. Once the power set functor in toposes was identified, it was clear that, as in the category of sets, it was representable by an object, the 'subobject classifier', or 'object of truth values'. It was this discovery which led to the concept of elementary topos, which Myles Tierney and I developed in 1969-70."

"I understand that your collaboration with Tierney took place in Canada at Dalhousie University, where you were one of the first recipients of a Killam professorship. This was evidently a very fruitful collaboration."

"Yes indeed, and in fact the Killam professorship enabled me to support a number of collaborators on the project, the major goal was the formulation of elementary axioms for the topos concept. Myles presented a weekly seminar in which the current stage of the program was described. Some of our work took the form of discussions in the seminar itself: remarks by participating students sometimes provided key steps in our work. . Although I had convinced myself that a finite elementary axiomatization for topos theory was possible, it took a number of successive simplifications to arrive at the compact list of axioms we now know are sufficient. The criterion of sufficiency was that, given any category satisfying the axioms, that is, an elementary topos, it should be possible to extend it by presheaf and sheaf methods. The presheaf aspect was covered by the 'fundamental theorem' that the category of objects over a given object in topos is itself a topos, together with the later discover that the category of coalgebras for a left exact comonad on a topos. The sheaf concept led to the conjecture that subtoposes of a topos would be precisely parameterized by certain endomaps of the subobject classifier, and this was verified. Those endomaps became known as Lawvere-Tierney modal operators, and correspond classically to Grothendieck topologies. The fact that the theory is elementary, that is, first-order, implies that it has countable models and other features making it applicable to independence results in set theory and to higher recursion."

"'In 1971,' I remarked, "you left Canada for Denmark, and the collective involved in the birth of the elementary topos concept was dispersed. What led to this?"

"Certain members of the collective, myself included, became active against the Vietnam war and later against the Canadian War Measures Act proclaimed by Canadian Prime Minister Trudeau. That Act, similar in many ways to the U.S. Patriot Act 35 years later, suspended civil liberties under the pretext of a terrorist danger. The alleged danger at the time was a Quebec group later revealed to be infiltrated by the Canadian secret police. Twelve communist bookstores in Quebec, unrelated to the terrorists, were burned down by the police; several political activists from various groups across Canada were incarcerated in mental hospitals, and so on. I publicly opposed the enforcement of this fascist law, both in the university senate and in public demonstrations. The administration of the university thereupon declared me guilty of 'disruption of academic activities'. Pernicious rumors began to circulate, for example, that my categorical arrow diagrams were actually plans for attacking the administration building, and possibly further outrages. I acquired the reputation of being a dangerous radical and as a result my Canadian contract was not renewed. After my dismissal I spent some time in Denmark and later Italy, a country for whose culture and people I developed a particular affection. In 1974 I returned to the United States, where I joined the mathematics department of SUNY at Buffalo. That I was able to do this was in no small measure due to the efforts of John Isbell and Jack Duskin in persuading the Dean that (contrary to the message sent out by one of the Dalhousie deans) I was not a danger and might even be an asset!"

"So you found a safe haven in turbulent times," I said. "But to return to category theory. You see category theory not only as a potential foundation for mathematics, but as a method of philosophical analysis, especially in dialectical philosophy. Would you enlarge on this?"

"Sure," Lawvere responded. "But let me begin with a few observations on philosophy in general, as I see it. Philosophy had the ancient and honorable role as a servant to the learning, development and use of scientific knowledge. In his Lyceum, Aristotle used philosophy to lend clarity, directedness, and unity to the investigation and study of particular sciences. The programs of Bacon and Leibniz and the important effort of Hegel continued this trend. One of the clearest applications of that outlook to mathematics is to be found in the neglected 1844 introduction by Grassmann to his theory of extensive quantities. Optimistic affirmations and applications of the idea are also to be found in Maxwell's 1871 program for the classification of physical quantities and in Heaviside's 1887 struggle for the proper role of theory in the practice of long- distance telephone-line construction. In the latter, Heaviside formulates what has also been my own attitude for the past thirty years, namely, that the fact that our knowledge will never be complete, and hence no general theory will be final, provides no excuse for not using now the most general theory which science can support, and indeed for accuracy we must do so. To students whose quest drives them in the above direction, the official bourgeois philosophy of the 20th century presents a near vacuum. This vacuum is the result of the Jamesian pragmatist trend clearly analyzed by Lenin in 1908, but 'popularized' by Carus, Mauthner, Dewey, Mussolini, Goebbels, et al. in order to create the current standard of so-called 'truth' in journalism and history. This trend has led many philosophers to become entranced by the thesis that true knowledge is impossible. Naturally this 20th century vacuum has in particular attempted to suck out what it can of the soul of mathematics: a science student naively enrolling in a course styled 'Foundations of Mathematics' is more likely to receive sermons about unknowability, based on some elementary abstract considerations about subjective infinity, than to receive the needed philosophical guide to a systematic understanding of the concrete richness of pure and applied mathematics as it has been and will be developed."

"It seems to me," I said, "a bit extreme to lump together Mussolini and Goebbels with the other names you mention. Mussolini was a political Fascist and Goebbels a rabid Nazi. This can hardly be said of the others, who, while espousing philosophical and social positions that, understandably, meet with your disapproval, would presumably have rejected Fascism and Nazism."

"Perhaps I exaggerated a bit with my inclusion of Mussolini and Goebbels," Lawvere responded. "But I do insist that the subjective-pragmatist view that true knowledge is impossible was shared, in various degrees, by all of the people I mentioned. But allow me to continue with what I was saying. While philosophy in this century has, in my view, stagnated and even regressed, this is not the case for mathematics. Mathematicians at their work benches have been forced to fashion philosophical tools (along with those proofs of theorems which are often allegedly their sole product), and to act as their own 'Aristotles' and 'Hegels' as they struggle with the dialectics of 'general' and 'particular' within their field. This is done in almost complete ignorance of dialectical materialism and often with understandable disdain for philosophy in general. It was the struggle with a problem involving spheres and the relation between passage to the limit and the leap from quantity to quality which led Eilenberg and Mac Lane in the early 1940's to formulate the general mathematical theory of categories, functors, and natural transformations. Similarly, study of concrete problems in topology, functional analysis, complex analysis, and algebraic geometry in the 1950's led Kan and Grothendieck to formulate and use important further advances such as adjoint functors and Abelian categories. And the past thirty years have not been devoid of progress: in the last few decades a panoply of categorical tools have been forged by numerous researchers with strong ties to very branch of mathematics. These tools include toposes, enriched categories, 2-categories, monads, indexed categories, synthetic differential geometry. In particular all the now-traditional areas of subjective logic have been incorporated with improvement into this emerging system of objective logic. I see in the ongoing development of category theory the emergence not only of a new and decisive way of advancing mathematical understanding, but also as providing the tools for a clarification of dialectical philosophy. Philosophy underlay the development of mathematics, now, through category theory, mathematics can drive the development of philosophy. It is my belief that the technical advances forged by category theorists will be of value to dialectical philosophy, lending precise mathematical form to ancient philosophical distinctions such as general vs. particular, objective vs. subjective, being vs. becoming, space vs. quantity, equality vs. difference, quantitative vs. qualitative etc. In turn the explicit attention by mathematicians to such philosophical questions is necessary to achieve the goal of making mathematics (and hence other sciences) more widely learnable and useable. "

"Well, it would seem that you see a bright future for mathematics," I observed. "I was intrigued by your remark that it is necessary for mathematicians to bring 'explicit attention' to philosophical questions. Could you give some examples of this in your own mathematical thinking?"

"Sure," Lawvere responded. "Two examples come to mind. In the early 1970s I began to view elementary toposes as categories of 'variable' sets, that is, as an embodiment of variability, or objective change. As Engels remarked in the period when set theory and the arithmetiztion of analysis did not yet dominate mathematical thinking, the advance from constant quantities to variable quantities is a mathematical expression of the advance from metaphysics to dialectics. I identified a parallel, but deeper, philosophical advance in the passage from the category of Cantorian 'constant' sets to general toposes of 'variable' sets. Then, In early 1985, while I was studying the foundations of homotopy theory, it occurred to me that the dialectical concept of the unity of opposites could be usefully represented by maps equipped with two sections, or equivalently, by pairs of isomorphic subobjects of an object with a common retraction . In the examples I considered it was natural to construe these subobjects as 'opposites' united by the common retraction. I found that this idea to provide an effective basis for teaching calculus along the lines suggested by Karl Marx."

"Could you explain this?"

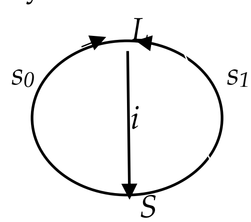
"Sure. But I'll need to write some symbols down." Producing a notebook and pen from his jacket pocket, Lawver continued, writing down the occasional formula, "As you no doubt know, before the development of theory of limits and the arithmetization of analysis the differential calculus had been plagued by a glaring contradiction in the process of calculating the derivative of a function. This arose from the dialectical opposition between constant and variable quantities. In calculating, for example, the derivative of the function $y = x^2$, one began by assigning the variable x two *different* values x_0, x_1 giving rise to the values $y_0 = x_0^2, y_1 = x_1^2$ for y. Then the quotient

$$\frac{y_1 - y_0}{x_1 - x_0} = x_1 + x_0$$

is formed. This quotient is well-defined because of the initial assumption that $x_1 - x_0 \neq 0$. So far so good. Then the derivative is defined to be the above quotient obtained by taking x_1 to be equal to x_0, that is, $2x_0$. As pointed out (and exploited for religious purposes by) Berkeley, this procedure contains an outright inconsistency. For while granting that the 'variable' quantity x is capable of assuming different values, in the first part of the calculation x is assigned the two *constant* values x_0, x_1 which are explicitly specified to be different. It is inconsistent, then, to take these values to be the same. To put it another way, the constant quantity $x_1 - x_0$ cannot simultaneously both be equal and unequal to 0. Towards the end of his life, Marx wrote about the foundations of the differential calculus. Although he was not a mathematician, he struggled with this basic contradiction (as had others before him). He saw that the difficulty arose from the opposition between constancy and variability. The essence of his line of thought is that in calculating the derivative of a function $y = f(x)$ the variable x is 'split' into the pair of *variables* x_0, x_1 with the constraint that the difference $\Delta x = x_1 - x_0$ is $\neq 0$. The calculation then proceeds under this assumption, and the difference $\Delta y = f(x_1) - f(x_0)$ is expressed in the form $\varphi(x_0, x_1)\Delta x$. The expression $\varphi(x_0, x_1)$ is what Marx called 'algebraic', that is, Δx does not appear in it, so that it is independent of Δx. In particular $\varphi(x_0, x_1)$ is not subject to the constraint $\Delta x \neq 0$, so allowing x_1 to be set equal to x_0 in it. The resulting expression $\varphi(x_0, x_0)$ or, dropping the subscripts, $\varphi(x, x)$ is then defined to be the derivative of $f(x)$."

"So what you are saying," I observed, "is that in his analysis of the differential calculus Marx used the idea of splitting a variable into two and later collapsing them to one."

"Yes, exactly," Lawvere continued. "And this process of splitting variables into 'opposites' and then collapsing them to one again collapsing variables is, as Marx saw, an instance of the unity of opposites. I saw the issue in the following way. Assuming those laws of algebra which are equally valid for variable and constant quantities, what is additionally required in order to determine the derivatives of genuinely variable quantities and to establish the laws of the derivative? It occurred to me that this whole problem could be illuminated by taking the given variable quantities *x* and *y* as living in one ring *S* (for 'smaller') and the 'split' variable quantities as living in a larger ring *L* containing 'more' variable quantities than does *S*. The 'splitting' procedure would be represented by two maps $s_0, s_1 : S \to L$ which can be thought of as 'opposites' and the collapsing of the two opposites by a map $i : L \to S$. The maps s_1 and s_2 are both sections of *i*. This setup is naturally visualized as a 'mapping cylinder'



Now, given *x* in *S*, let Δx stand for the difference $s_1(x) - s_0(x)$ in *L*. Δx represents the variation in *x* when it is converted into the opposite quantities $s_0(x), s_1(x)$. For any quantity in L, let us call its collapse its image in *S* under *i*. The collapse of the variation in any quantity in *S* is always 0. That is, collapse negates variation. Now we can define a quantity *x* in *S* to be *variable* if its variation Δx does not divide 0 in the sense that, for any quantity *q* in *L*, $q.\Delta x = 0$ only if *q* collapses to 0 in *S*. Given two variable quantities *x* and *y*, let us say that the variability in *y* is dependent on that of *x*, or simply that *y* depends on *x*, if Δy is a multiple of Δx by some multiplier in the larger ring. These multipliers are naturally thought of as 'secant-slope' functions. Then, if *y* depends on *x*, all these multipliers have a single common collapse in the smaller ring which can be identified with the derivative *dy/dx*. Defined in this way, the derivative is naturally thought of as a 'tangent-slope' function. The basic laws of the differential calculus are now easily derivable. This provides a satisfactory basis for teaching calculus, since it codifies the basic idea: the suitable categories are those in which the secant-slope function always exists and specializes unambiguously to a tangent-slope function. The essential point is that the contradiction in the traditional calculation of the derivative takes place in a *single* ring S, the ring of real numbers. The contradiction is resolved by introducing a *second* ring L dialectically linked to S through a mapping cylinder. Since mapping cylinders can be interpreted in arbitrary categories, this approach provides a basis for extending the differential calculus thereto."

"That's quite fascinating, and I shall have to think about it." I said. "Now I understand that you are very concerned with mathematical education, and especially the teaching of the calculus. Would you care to elaborate further on this?"

"Sure," Lawvere responded. "Teaching the calculus to students in a way that is both practical and explains the underlying principles in a cogent manner is a major issue in mathematical education. For a long time calculus teachers have been demanding better textbooks. In the past few years, after much fanfare, a few books claiming to respond to this demand have been launched. But many teachers are saying already that these 'new' textbooks are very similar to the old ones that worked so poorly, except that the 'new' textbooks contain many elaborate and subtle attempts at persuading the student (and the teacher) that some kind of 'understanding' is going on, even while all the more resolutely concealing the essential principles.

"Also, a journal for the guidance of college math teachers carried a lead editorial aiming to refute what it called the 'myth of scientific literacy'. Under the banner of this 'scientific literacy', the past few decades have seen a major restructuring of the scientific curriculum. This restructuring involved what scientists judged to be a forced dilution of content, aimed at producing students with no genuine understanding of science, but who would be able to speak and write about it all the same. But this already deplorable situation has recently degenerated still further, for we are now told that most students should not be required to attain even this 'literacy', but merely a diminished goal of 'awareness'. A current example (from the journal of a teachers' union) of what such awareness means is the ability just ton recognize terms such as 'chaos theory' and 'paradigm shift' and on that basis submit to whatever post-modernist reorganization is being proposed. As if this were not enough, the claim that we are now in a 'postmodern' and 'playful' age is being put forward, by university centers for pedagogy, as a basis for replacing the learning of mathematics with 'an inquiry into our habits'. In an attempt to enlist professors for this transformation, the old ecoterrorist claims against mathematics have been resurrected- that mathematical thinking supports military technology and destruction of the environment, and that we 'are' our mathematics. A pseudo-philosophical basis for these counter-reform 'reforms' is also being provided. In U.S. and Canadian journals for college math teachers, it is revealed that mathematical ideas are social in character and that social rituals constitute a third category of being. We are also assured that the very notion of a curriculum constitutes a barrier between the individual and the collective. Certain mathematics publications have been fulsome in their praise of the philosophers Kuhn, Lakatos, Feyerabend, and Popper, one saying that the philosophy of mathematics needs some great innovators in their mould; another calling them exemplary 'philosophers of Irrationalism '."

"It seems absurd," I interjected, "to call all of these philosophers 'Irrationalists'. That may have been true in Feyerabend's case, but he was probably just trying to *épater le bourgeois.* Although Popper was known for his theory of falsification, and for his criticism of Marx, he was fundamentally a realist and Lakatos began as a dialectical materialist but later became a follower of Popper. In Kuhn's case his claim that science does not evolve gradually towards truth but undergoes sudden 'paradigm shifts' would certainly not meet with your approbation, but can scarcely be called a form of 'Irrationalism'."

"It was not I, "Lawvere rejoined, "but the mathematical publication in question which called them 'philosophers of Irrationalism' . My point here was that their philosophies, while differing in detail, are all sufficiently ant-scientific for them to be praised by propagandists for their 'Irrationality'."

"Very well," I said. "Let us return to mathematics education. Do you regard what you see as the current degeneration of mathematics education and philosophy as the product of a long-term trend?"

'Yes, I do," Lawvere replied. "The current relation between teaching of mathematics and philosophy of mathematics is, I am convinced, just the latest stage in a long process. Three hundred years ago the Leibniz rule for the rate of change of a product, and Newton's theorem that the rate of change of the area under a curve is the height of the curve, were made explicit and publicized. This calculus was developed by Bernoulli and Euler, Cauchy and Maxwell into the universal instrument for designing engines, ships, electric power and communication systems, etc. Yet after three hundred years, most people, including many who actually build those wonders, are still in no position to challenge these designs on their own grounds because knowledge of the instrument has been denied to them. Even more bizarre is the lack of comprehension is shared by most people who have been through courses which the State has gone to considerable effort to provide."

"Yes, that last fact is very striking," I said. "It isn't surprising that the calculus is unintelligible to the public at large, given the incomprehension and dislike of mathematics in general. But it is strange that many people who have received actual instruction in the calculus don't really understand it either."

"It isn't strange at all," Lawvere rejoined. "It's a planned consequence of the program by so-called progressive 'educators' of replacing genuine understanding of objective reality by the personal gratification produced by social acceptance. This goal was enthusiastically endorsed by the unfortunately numerous professors who believe that most students are incapable and unwilling to learn any serious subject. But already around 1750 in Milan, Italy, Maria Agnesi wrote and printed a textbook, based on the premise that all Italian youth could and should learn calculus. Her enlightened vision is yet to be realized (in Italy or elsewhere)."

'Why is this, do you think?"

"After the decimalized system was introduced during the French Revolution, it became necessary to recalculate the trigonometric and logarithmic tables used in construction, navigation and other technical activities. A few years ago the following true story about this was widely publicized by a giant multinational computer corporation. An accomplished engineer, Gaspard de Prony, was appointed to organize the large task of calculation. Borrowing from Adam Smith, he divided the personnel into three levels: Level A consisted of a few mathematicians who were able to invent appropriate formulas. Level B comprised a somewhat larger group of people who were able to convert the formulas into algorithms. Finally, a much larger group C, were employed to actually carry out the algorithms by adding and multiplying. The point which the computer corporation found worthy of resurrecting two hundred years later was this: 'It was found that the work went more smoothly if those in group C knew no mathematics.'"

"Ha!" I exclaimed. "An irony indeed. Do go on."

“In Britain in the 1830's, Lawvere continued, “millions of people were in motion, demanding democracy. Among the measures for quelling the demands of this Chartist movement, the Privy Council created for the first time a system of state-supported schools and teacher-training and inspection, in order to ‘introduce order and discipline into the working-class population when older methods of wielding authority had broken down.’ The education provided by these new schools was consciously two- tiered: for exampl , one of the defining documents states that ‘Arithmetic is the Logic of the poor’. In the school systems modeled on that idea, it is clear that while teachers are struggling to teach, administrative expenditures and regulations have the larger aim of insuring that not too much is actually taught. “

“‘Ignorance is Bliss’ elevated into a pedagogical principle, no less.”

“Yes, exactly. Still, In the period before 1848 there was optimism about the possibility of general enlightenment. For example, the Danish physicist Oersted, who discovered an important principle relating electricity and magnetism, set up an institution to make it known to all. The German mathematician Grassmann, who in 1844 published a new theory and method in geometry which now is becoming widely used by physicists, was in fact a high-school teacher who insisted that his new dialectical philosophy was at least as important, since it was directed explicitly at assisting students to learn and understand. However, in the 1870's, when one of his followers published a book showing in detail how Grassmann's methods could be used to teach not only geometry, but also to introduce calculus in high school, he received a very scathing review and condemnation for suggesting such an upset in the Prussian order of things. The author of the review, Felix Klein, was later the official representative of that Empire at the World's Fair held in conjunction with the opening of Rockefeller's University of Chicago.”

“I did not know that Klein had played that role,“ I said.

“He did indeed,” Lawwere continued. “Anyway, following the Privy Council's lead, a body of technique was created, a sort of prize-fighter's technique for occasionally appearing to give in to the demands for reform while actually thereby directing teachers’ energies to serve an opposite aim. For example, forty years after Grassmann's death the Prussian establishment decided to make him ‘a great German soul’ but he was portrayed in pragmatist journals as a philosophical idealist. In 1908 Lenin defended Grassmann's materialist philosophy from this unwarranted distortion, and remarked, in connection with certain proposals to introduce higher mathematics into the schools, that it was surely not being done in order to deepen and broaden the knowledge of science, but rather to provide a basis for the promotion of idealist philosophy.'

"Higher mathematics paving the way for idealist philosophy, in effect," I said.

"Yes indeed," Lawvere rejoined. "In fact, the organizers of collegiate mathematics teaching and the popularizers of pragmatism, with its instrumentalist conception of truth, were closely associated for many years, and leading circles of philosophy, such as the Gifford Lectures in Scotland and the Silliman Lectures in Yale, began to systematically misuse mathematics and especially their audience's ignorance of mathematics. The Prime Minister of the British empire (nicknamed 'bloody Balfour' for his suppression of the Irish, later famous for his declaration in support of Zionism), who wrote several books on philosophy, was also known for his Education Act which reorganized the high schools. Balfour stated in one of his Gifford lectures 'I wish I were a mathematician'. Being known as a mathematician became a route to historical recognition as a philosopher. For example , Bertrand Russell's opinions on everything became sought-after and he even eventually received the Nobel Prize for Literature, partly because of his notoriety as a mathematician. Through clever wordplay he devised a new branch of philosophy known as 'foundations of mathematics' whose only role is to give mathematics permission to exist, and which must be written in symbols different from the usual mathematical ones. This ruse he had learned from Peano, whose followers had proudly produced a high-school text written entirely in symbols, in order to dispel any false idea that with 'numbers', 'lines', or 'space' we are really referring to anything; amazingly, this work was advertised as a clarification of Grassmann."

"I can't agree with you about Russell's achievements," I protested. "Russell was deservedly awarded the Nobel Prize for Literature for his sparkling writing style and the range of philosophical and socio-political topics he wrote about. I think the fact that he was a mathematician had little to do with it. Also I think your assertions unfair that his work on the foundations of mathematics was based on mere 'clever wordplay' and that its sole purpose was to give mathematics 'permission to exist'. Its purpose was to show that mathematics had a logical character and that its propositions expressed actual truths about the world. The purpose of the admittedly esoteric notation he introduced was not to obscure mathematical notions that were already clear, but rather to give precise expression to what he saw as their logical character. I can see why you would dislike Russell's approach, since it emphasized what you have called subjective, rather than objective, logic. Yet Russell himself thought that the logic he was dealing with was objective, since it was supposed to underpin objective truth. He was very far from being a pragmatist."

"Well, we will have to agree to disagree about Russell," Lawvere rejoined. "While he was certainly a sparkling writer, and an atheist, I think that his approach to the foundations of mathematics was entirely wrong-headed. I have to admit , though, that you have put your finger on my fundamental objection to Russell's approach, namely that it was based entirely on subjective logic and, whatever Russell himself may have believed, not at all on objective logic."

"All right," I said. "Let us return to the pragmatist movement. You see it as having a major, and pernicious, influence on education, and mathematical education in particular."

"Yes, without question, " Lawvere replied. "Perhaps the best-known 20th century figure whose actions were consciously guided by the pragmatic philosophy was Mussolini – yes, Mussolini again! - but probably as important was the pragmatist John Dewey, who, while, not a Fascist, through his teachings and organizations had a tremendous influence on education throughout the world. He was occasionally quite candid about the direction of his reform; for example, in China in 1919 he gave a course at a college for teachers, in which he enunciated his infamous principle: 'Teach the child, not the science!' But of course conscientious teachers through the centuries have done both: the acquisition of some portion of the accumulated knowledge of humanity (science, that is) is the purpose of the child's presence in school, but the teacher endeavors to guide this acquisition with due regard for each child's particular situation. Why then Dewey's prohibition of the teaching of the knowledge? In China he compared the alleged 'authoritarianism' of science with the recently-overthrown imperial regime, and since then the broad-brush charge of 'authoritarianism' has been used thousands of times as a pretext to eliminate from school systems the teaching of the deductive aspect in geometry, of the grammatical parts of speech, of the diagramming of sentences, etc. Indeed, many college teachers of mathematics have now recognized that in large part the mathematical difficulties experienced by students fresh from high school are not due to mathematics itself, but are the result of their first real encounter with the requirement that ordinary language be used in a precise way. Dewey's powerful principle of 'teaching the child, not the science' has many corollaries, such as the anti-child theory that 'learning is fun', and ultimately the replacement of reasoning by jokes. Certainly the principle includes the injunction often addressed to pupils: 'Say it in your own words'. This injunction is very attractive to teachers, who know that understanding requires a conscious act by the individual. However, the whole atmosphere of the school often mandates that 'in your own words' should mean 'as imprecisely as possible', thus destroying the acquisition of concepts in any usable form."

Lawvere paused again to light a cigarette and then resumed: "In the United States the tradition that the purpose of schools is the imparting of human knowledge was being eroded in another, equally pernicious way. Early in the 20th century the steel city of Gary, Indiana was built at the decree of U.S. Steel. This included the factories, workers' homes, sidewalks, as well as the school system. To minimize the free time of the sons and daughters of the workers, extra-curricular activities at the schools were declared essential to the development of 'the child'. This Dewey-endorsed school system was studied by administrators from all over the world who journeyed to Gary to observe it in action. The whole Dewey program was styled 'progressive' education, illustrating through the use of this term the fundamental tenet of pragmatist epistemology: truth is what you can get away with."

"And what was the effect of this 'progressive' program on mathematics education?"

"In 1915 the U.S. mathematical organization split into two, one devoted primarily to the promotion of research, and the other supposedly devoted to the promotion of college teaching. The latter maintained and deepened its ties with pragmatic philosophy in 1921 when, at a meeting at Wellesley College, the widow of the leading publicizer of pragmatism, Paul Carus, whose stated aim was to promote religion on the basis of recent science, gave several thousand dollars to finance a series of monographs. At the same meeting the president of the organization, the author of one of the few texts on the history of mathematics then available in English, gave an address entitled *Religio Mathematici*, in which he proclaimed such enlightening principles as 'since we know infinitesimals, we must also know our own insignificance; since we believe in infinity, we must also believe in an Almighty; since we can imagine the fourth dimension, we can also imagine heaven, etc, etc.' Carus edited the philosophical journal *The Monist* from 1890 to 1919; its journalistic policy was to exploit recent scientific results (not yet widely understood by the public) to cast doubt on science and thus to rescue religious speculations from the advance of science. The method used, borrowed not unwittingly from Bishop Berkeley, led to a tortured definition of 'science' that permitted Carus to exult after the World Parliament of Religions held in Chicago in 1893, that Buddhism is the 'most scientific' of religions. His name is well known to mathematicians as the title of a series of expository monographs, the Carus Monographs, issued by the Mathematical Association of America. That series has been self-supporting for most of its life owing to the mathematical and pedagogical virtues of its contents, which were real despite the obscurantist purpose underlying the publication of the series. The purpose of Carus's organization has never been repudiated, and its publications, which aim to give guidance to college teachers, have over the years refined to a precise art a writing style similar to that of the Scientific American, in which it is presumed that readers will not advance from a lower to a higher level, and hence under the guise of 'popularization', all concepts are rendered sufficiently imprecise so as to be unusable by anybody."

"Still," I felt obliged to point out, "mathematics itself has made spectacular progress in the 20th century, as you have already acknowledged."

"Of course," Lawvere responded. "It's often remarked, correctly, that mathematics has made more advances in the 20th century than in all previous centuries. These advances include not only the formulation and solution of difficult problems with geometrical and other content, but also (indispensably to that) the development of unifying concepts which are of great simplifying and clarifying value. Abstract algebra and category theory are outstanding examples. Unfortunately, this progress in mathematical science has not led to a corresponding simplification and clarification in the teaching of mathematics. In this regard an opportunity seemed to present itself around 1960. The occasion, as I understand it, was the following. The U.S. ruling circles, fresh from rejoicing that their friend Khrushchev had succeeded in overthrowing the socialist system in the Soviet Union and was in the process of transforming it into a pseudo-socialist system, suddenly realized that they were thereby also faced with a rival superpower. This implied a certain shift of the boundary between the B and C Levels on the Prony scale I have already mentioned, as well as a readjustment of the line between 'arithmetic' and 'logic' on the Privy Council's anti-Chartist plan I have also mentioned. It meant that more students would have to learn more math and science in order to counter the Sputnik threat. Whatever the precise details of the background, around 1960 the opportunity arose to have university researchers directing summer schools for eager high- school teachers, to have writing teams producing new text books for pupils and teachers, etc. The challenge was taken up enthusiastically by many professionals in the spirit of allowing those concepts, which had they had found so enlightening, also serve to enlighten everyone. Of course, for such an undertaking to succeed it requires several years of pupil feedback and text revision (and new mathematical research!). But that stage was never reached because the movement was discredited; the enthusiastic professionals had underestimated the preparation of the opposition. “

I interjected: “By ‘the movement’ do you mean the ‘New Math’ movement of the late 1960s? The approach in which, according to Tom Lehrer, ‘the important thing is to understand what you're doing, rather than to get the right answer’."

“The ‘New Math’ program,” Lawvere replied, “was part of this progressive movement in mathematics education. As a satirist, Tom Lehrer’s intention in his remark was no doubt to ridicule the New Math, but actually its content was quite correct, for the primary goal of the progressive movement was indeed to emphasize the understanding of mathematical concepts rather than the process of arriving at the ‘right answer’ through blindly following rules of calculation. Of course, the ability to calculate is not incompatible with understanding the rules used in actually performing the calculations. Unfortunately, the New Math approach placed too much emphasis on purely logical and set-theoretic notions, which mystified parents who were ill-equipped to help their children with their mathematical homework. Then, by an artful confusion of the meanings of words like ‘foundation’, the foundationalist trend insisted that the texts must be written in their idiosyncratic notation. And the professional schools of pedagogy (housed since Dewey in ivory towers remote from the actual scientific departments) took leadership of the movement from the bewildered scientists, thereby ensuring its destruction.”

“How do you think mathematics education could be improved at the present time?” I ventured to ask.

“It would seem,” Lawvere responded, that an obvious way to improve math teaching would be to give more examples and more applications. But merely to demand this and then stop there is again to underestimate what we are up against. In the case of the teaching of calculus, the 1970s and 80s saw the publication of many ‘applied’ calculus texts in which explicit principles were subordinated to problems from various fields diluted and distorted beyond usability. But as many professors in those various fields understand, math is theory. What a student needs in a field of application such as chemistry, business management, etc. is to know math as well as possible in order that the applied concepts be approachable with little math-related mystery and in order that mastery of appropriate new methods can be partly self-guided. Isolated, particular methods learned mechanically and then forgotten, and, especially, half-baked attempts at teaching an alleged application instead of the explicit principles of calculus, can only negatively affect the students' ability to apply math. The demand for better calculus textbooks in the English-speaking world thus began, for these and many other reasons, to become more insistent. The initial response of the publishers - that they would never change their policy of offering the next year an exact copy of the competing text which had made the most profit the previous year, only with more colors - was met with well-deserved contempt on the part of teachers. Some of the more recent offerings are the result of multimillion dollar government intervention."

"What do the university professors of mathematics think of all this?" I asked. "For example, do they see a gap between teaching and research? The common term 'teaching load' used in universities would seem to indicate that professors regard teaching as a burden rather than a joy."

"Contrary to the portrait of the professor who views teaching and research as inimical to each other, many see the two as mutually supportive. Many of the ideas which have led to long and fruitful development by researchers actually arose from attempts to explain matters more clearly to students. For example, attempts in the 1960's to provide a clearer, simpler, yet rigorous base for understanding calculus led to a new trend of research in the foundations of topology, logic, and analysis within which many innovative papers and over a dozen books have now been produced. On the other hand, research leads from time to time to new synthesizing concepts, which clarify matters enormously for the researchers, who then struggle to find ways to disseminate this clarification to students. For example, research into the mathematical foundations of continuum mechanics and the constitutive relations of materials have led to new, more direct, ways of dealing with infinitesimals, function spaces, and extensive quantities, which are now being taught to undergraduates in certain places. Research and teaching are of course different aspects of endeavor, but as long as they are still alive, they have an orientation in common, a commitment to tirelessly combating the absence of knowledge. History shows that the teachers, yearning for a greater opportunity to participate in the creation and dissemination of enlightenment, will not be satisfied by waiting for this or that establishment entity to provide it. Not only would waiting cause the fulfillment of these needs to remain forever a mere policy objective, our enthusiasm would continue to be used as the engine for the spread of still more pseudo- knowledge and pessimism. The problem can be solved, without million-dollar grants, both by devising teaching materials which reflect the actual historical development of a given field - neither repeating some entrenched hundred-year old false summation, nor succumbing to the ultrarevolutionary post-modernist degeneration - as well as by making explicit the philosophy which emerges from actual research developments of recent decades. Collective effort is necessary, however, to concentrate such materials and to disseminate them so as to serve the needs of society as a whole."

"A pedagogical call to arms, then!" I exclaimed.

"Yes indeed," Lawvere returned. "Of course this call to arms, as you correctly put it, will require philosophers to learn mathematics and mathematicians to learn philosophy!"

"Do you think that this is likely to happen?" I as ked.

"Well, meetings like the present one in which both philosophers and mathematicians are involved gives me hope that it will!"

"Yes, it is a hopeful sign," I said. "Now you have mentioned Grothendieck a number of times. What are your memories and impressions of him?"

"Of course by the middle 1960s Grothendieck had already acquired a legendary reputation among mathematicians and his work had a major impact on many of us, myself in particular. I had my first encounter with him at the 1970 ICM in Nice, where we were both invited lecturers. I publicly disagreed with some points he made in a separate lecture on his 'Survival' movement, so that he later referred to me (affectionately, I hope) as his 'main contradictor'. In 1973 we were both briefly visiting Buffalo, where I vividly remember his tutoring me on basic insights of algebraic geometry, such as 'points have automorphisms'. In 1981 I visited him in his stone hut, in the middle of a lavender field in the south of France, in order to ask his opinion about a mathematical project of mine. He saw right away that to carry my project through successfully would require the use of the subobject classifier in a topos, which, as he said, is one of the few ingredients of topos theory he had not foreseen. Later in his work on homotopy he kindly referred to that object as the 'Lawvere element'. My last meeting with him was at the same place in 1989. He was clearly glad to see me but would not speak, the result of a religious vow; he wrote on paper that he was also forbidden to discuss mathematics, though quickly his mathematical soul triumphed, leaving me with some precious mathematical notes. But the drastic reduction of scientific work by such a great mathematician, caused by his encounter with a powerful designer religion, is cause for renewed vigilance."

“You have mentioned your public disagreements with Grothendieck. Could you describe them?”

“The position taken by Grothendieck which formed the ideological core of his ‘Survival’ movement amounted to the wholesale rejection of science (including mathematics) on the grounds that it has become a religion whose ‘high priests’ are the scientists themselves. He claimed that this new religion, which he called ‘scientism’, has largely replaced the older traditional religions but is, in its way, equally dogmatic and faith-based, in its case on the experimental-deductive, or ‘scientific’ method. Much as I admired, and scontinue to admire Grothendieck’s genius as a mathematician, I felt I had to make my objections to his traducing of science and mathematics public.”

I responded, “I can understand why, as a dialectical materialist and a champion of the scientific world-view, you would reject Grothendieck’s identification of science as a religion. But from my reading of Grothendieck’s pronouncements in the late 60’s and early 70s, my feeling is that his major objection to the pursuit of science was that it was blindly leading humanity to disaster. I’d guess he felt that the ‘miracles’ of science such as nuclear weapons and computers would not be mere myths as in previous religions – harmless, if absurd ones such as Jesus’s conversion of water into wine - but could, and probably will have destructive consequences on a vast scale. In this respect Grothendieck seems to me an avatar of the ecological movement. Correct me if I am wrong, but I doubt that you have objections to that aspect of Grothendieck’s thinking.”

“No, Lawvere responded, “I think Grothendieck ‘s warnings about the dangers of the misapplications of science are both timely and fundamentally correct. He rightly detested the nefarious purposes for which the ruling class has used science. But the ‘science’ he rejects is no more than a distorted version of genuine science produced by the functionaries of that ruling class, distorted precisely in such a way as to enable it to play the role of a new religion, enshrining through its apparent objectivity the authority of the ruling class. The distortion here arises not through the dissemination of empirical falsehoods - even this distorted version of science has not yet degenerated to the level of the politics of the ruling class! - but is a subtler deformation arising through the cynical use of science as an instrument of power by that class, which has little interest in truth as such. “

“I would surmise,” I said, “that one of your principal objections to Grothendieck’s views is the emphasis he places on subjectivity, on emotion and feeling which because of their private nature cannot be objects of public scientific investigation. Is this surmise correct?"

"Well," Lawvere replied, "I've already told you when you brought up Husserl earlier that I refuse to wallow in subjectivity! But you are right, I do think that Grothendieck's overemphasis on the subjective is central to his critique of science. And, sadly, that overemphasis created an obsession with his own consciousness, an unhealthy hypersubjectivity, that led him first to fall into the clutches of various designer religions, and later into what seems to be a form of mental derangement. Hypersubjectivity makes a person vulnerable to such slippage, and, I'm sorry to say, Grothendieck, a truly great mathematician, was tragically vulnerable in this regard."

At this point I thought that our discussion should be brought to a conclusion. I said "I'd like to end with a final question. I've noticed that in your writings you often say that a branch of mathematics, or a particular mathematical theorem is 'discovered' rather than 'created' or 'invented' or 'proved'. For example, you refer to Grassmann as the 'discoverer' of linear algebra, to Eilenberg and Mac Lane's 'discovery' of categories and functors, Kan's 'discovery' of adjoint functors, etc. To me the word 'discovery' connotes the revealing of something already in existence but hidden, as for example Pythagoras' discovery of the correspondence between euphony and simple ratios of string lengths, or Kepler's discovery of the form of the planetary orbits as expressed in his three laws of motion. On the other hand, it would take an extreme Platonist to claim that Michelangelo 'discovered' the statue of David in the block of marble or Beethoven 'discovered' the Ninth Symphony in the diatonic scale. What is discovered exists before its discovery, but what is invented does not exist before its invention. Could you tell me what you have in mind when you use the word 'discovery' in mathematics?"

"Ah yes, the old issue of discovery versus invention!" Lawvere responded. "As you say, what is discovered exists before its discovery, but what is invented, or created, does not exist before its invention or creation. The one, a being still subject to further becoming, the other, a becoming not guaranteed to achieve full being. This is a perfect example of a dialectical opposition. The opposition between discovery and invention is analogous to the opposition between the objective and the subjective. Considered as concepts, discovery and the objective are linked by the fact that only what is objective can be discovered. Invention and the subjective are likewise related by the fact that only what is conceived in subjectivity can be invented or created. So when I wrote that Grassmann discovered linear algebra it was implicit that linear algebra had an objective content, a content transcending the purely subjective, even though Grassmann's discovery was necessarily, as with all thinking, made within his consciousness. All true 'discoveries' in mathematics and science have this quality of arising within the subjective, and then rising therefrom to the objective. The purpose of science is to transform the subjective into the objective. The purpose of art, on the other hand, is to expand subjectivity itself. While this is an equally honorable goal, the subjective character of art cannot be expected to be enlisted in the advancement of our understanding of objective reality."

I got to my feet and said, " A truly deep note on which to conclude our discussion. Professor Lawvere, our conversation has been most enlightening. If I may say so, you are one of the very few mathematicians I have met who really takes philosophy seriously, and sees an intimate bond between it and mathematics. Thank you very much for talking to me."

Lawvere also rose and said: "You are most welcome, sir. It has indeed been an interesting conversation."

We bade each other farewell and went our separate ways.

Ludwig Wittgenstein

I had long wanted to talk to the famous philosopher Ludwig Wittgenstein but had not been able to contrive a way of getting to see him, even with the Liberator. Then fate took a hand…

In 1940 I returned to my old Cambridge college, Trinity, to attend a college feast. At Trinity High Table, in the midst of the dons assembled for dinner, resplendent in their dinner jackets, gowns and decorations, I found myself seated next to a man whose informal attire, tieless and gownless, contrasted strongly with that of the others. His sensitive teatures bore what struck me as a somewhat pained expression. This, coupled with the casualness of his dress, gave me the feeling that he was uncomfortable with the formality of the occasion. This intuition arose naturally in me because, along with the nostalgia I felt for my long vanished student days which had driven me to return to my old college, I recalled my feelings of absurdity at the formality of dining in college hall. These feelings began to recrudesce in the form of an irreverence that I wished to share in some way with my unconventionally garbed dinner companion. After I had imbibed a few glasses of one of the excellent wines that Trinity was known to serve at High Table dinners, I felt sufficiently emboldened to risk an attempt at cmmunication. The pocket recorder which I always kept with me took down the exchange that followed:

JB. *Well, what are we doing here, sitting in the midst of all this splendour?*

Dinner Companion (after an extended silence): I *don't know why we are here, but I'm pretty sure that it is not in order to enjoy ourselves. In my case, I' m certain of it.*

JB. *I'm sorry to hear that. Still, allow me to introduce myself. My name is Joel Bennhall. Many years ago I read mathematics as an undergraduate at this college. And you are, sir?*

DC. *I am Ludwig Wittgenstein. I am ashamed to admit that am a professor of philosophy in this university.*

JB. *Ah yes, I've heard of you! In fact, I've read a book of yours, the* Tractatus Logico-Philosophicus*. I think it was in 1923. It made a striking impression on me as as work of remarkable compression, clear as crystal in parts, yet baffling in others. It seemed to me that in writing it you set out with the intention of reducing philosophy to the expressible, but in the end washed up on the shores of the ineffable. I recall thinking that the first three-quarters of the work* *was a kind of preesentation of the world in monochrome. Only in the last quarter do you let the colour in, but, as you so wisely observed, that can only be shown, not said. "Whereof one cannot speak, thereof one must be silent" was, I thought, a splendidly dramatic assertion with which to conclude your work. But perhaps ending the work with of a number of blank pages might have made your point even more effectively by* showing *what you meant rather than* saying *it! I very much liked the form in which it was wriiten, which reminded me of Lichtenberg's* Sudelbücher. *Although your work can hardly be described as a "rough draft"!*

LW. *You seem to have understood the book better than most of my fellow-philosophers. But from that it's easy to infer that you are not a profesional philosopher! And yes, I have always admired Lichtenberg for the pithiness and content of his aphorisms. His example was in the back of my mind when I wrote my book. Even with Lichtenberg's help, writing it was a struggle, I can tell you!*

JB. *Perhaps a glass of wine would ease the memory of your struggle.*

LW. *Drinking wine and self-indulgence in general are essentially frivolous activities, but engaging in them helps people to live in the present, which makes them, and myself in particular, happy. So yes, a glass of wine would be welcome. Prosit!*

JB. *Yes indeed, Prosit, the German toast. You are Austrian, I believe?*

LW. *Yes, originally Austrian, although I have recently acquired British nationality, a necessity I could not have have envisaged in my youth in Vienna, even though I acquired some command of the English language in my youth. You mentioned that you had studied mathematics as a student here.*

JB. *Ah yes, during days long past, when I thought seriously of becoming a mathematician.* Aut Gauss aut nihil! *As an undergraduate here I was a contemporary of Bertrand Russell, whom I came to know quite well. Of course, we were both very young then! Judging from the Preface he wrote to your book, you must also have known him.*

LW. *Yes, I did know Russell well, and for a considerable time he exerted a powerful influence on me, but our views have come to deviate markedly in recent years.*

JB. *Did this deviation include your views on mathematcs? Russell proclaimed that mathematics was logic, and he and Whitehead wrote the massive* Principia Mathmatica *to prove this claim. What is your opinion of the idea that mathematics is logic?*

LW. *When I was young I was attracted by Frege's and Russell's account of mathematics as logic. They considered* truth *to be the essence of mathematics, and mathematics to consist of true propositions like 2 +2 = 4. But if one holds this view, one has to ask: what are true mathermatical* *propositions true* of*? Of coiurse, the traditional answer is: arithmetical propositions are true of numbers,geometrical propositions of geometrical objects such as points, straight lines and circles, etc. But this traditional answer came into question after mathematics expanded to include analysis and algebra. It is much more difficult to pin down what true propositions in these newer areas of mahtematics are true of. The inability of mathematicians and philosophers alike to provide satisfactory answers to these questions and their determination at the same time to extend the traditional conception of mathematical truth, with its associated certainty, to these new areas eventually led to the idea (already proposed by Leibniz) that mathematical truth is a form of* logical *truth, the form of truth that holds universally and is independent of subject matter. To establish this claim, it became necessary to cast mathematical propositions into logical form, and then show, using the laws of logic, that true mathematical propositions are nothing more than highly elaborated tautologies. Frege attempted to establish this claim for arithmetic, and in* Principia Mathematica *Russell and Whitehead tried to perform the same "service" for the whole of mathematics.*

JB. *Yes, I see. But it seems pretty clear that by the time you came to write the* Tractatus *you no longer held this view, if indeed you ever did.*

LW. *Yes, that's right. I had come to the conclusion that pure mathematics is not concerned with truth at all in the traditional sense. Rather, it is concerned with* correctness, *the correct appllication of* rules *laid down in advance. So, for example, one should say that 2 + 2 = 4, considered as a proposition of pure mathematics, is correct rather than true, since it is the result of the correct application of arithmetical rules such as associatity of addition, etc. As children we learn mathematics just as we learn to sperak and write, through the assimilation and observing of rules, with no mention of the idea of truth. The concept of truth emerges long after one learns to read, write, and calculate, and the only rules governng that concept are logical rules, e.g, p is true and q is true, then p & q is true. But I didn't see how the idea of truth can be reduced to a system of logical rules of this kind. This is why when I wrote the* Tractatus *I put forward the idea that a proposition is a kind of picture, and its truth amounts to an objective correspondence between it and what it depicts (so one might say that the picture is accurate) This correspondence – truth - is not given in terms of rules. It is just a fact, like the fact that only round pegs fit into round holes. But the remarkable thing is that, for so-called logical propositions, being true in the sense of* always *corresponding to the facts can be shown to be equivalent to derivability through corect application of the rules of logic. For logical propositioms, truth and correctnbess coincide. But in the case of mathematics there are no "facts" enabling the truth of pure mathematical propositions to be defined as "correspondence with the facts". Pure mathematical propositions cannot be true or false in the way that we understand "ordinary' propssitions to be. They can only be correct or incorrect. Of course, propositions of applied mathematics, such as "2 apples plus 2 apples = 4 apples" continue to be true (or false) in the familiar sense.*

JB. *I recently read an article describing the "current debate in the foundations of mathematics". As I recall, the author identifies thrree "schools"on the matter that have emerged recently which he calls Logicism, Intuitionism and Formalism. The first defends the thesis that mathematics is, in the last anlysis, logic. Intuitionism is the doctrine associated with Brouwer driven by the conviction that mathematical concepts and arguments are admissible only if they are adequately grounded in intuition. Finally, Formalism is the idea, which has come to be associated with Hilbert, that mathematics can be likened to a game played with symbols according to certain rules. It seems to me that your view that mathematics amounts to the correct application of rules as opposed to the pursuit of absolute truth is closest to Hilbertian Formalism in spirit.*

LW. *Yes, I agree. Hilbert had the insight that, when mathematics is stripped of the content usually ascribed to it, namely that it embodies truths about "mathematical objects", what remains is a system of symbols subject to purely formal, syntactical operations, rules, if you will. In this sense mathematics can be considered a game in which symbols are manipulated in accordance with rules whose application never requires appeal to the meaning of the symbols, that is to their extra-mathematical application. But I suspect that Hilbert is not truly a formalist. I think that in his heart of hearts he retains an attachment to the myth that mathematics has a "content", that it is descriptive of an objective reality which exists independently of how it is described. After all , was it not Hilbert who declared that "we shall never be expelled from the paradise that Cantor has created for us"? As far as I am concerned, "Cantor's paradise" is not a paradise at all and we should hasten to leave it of our own accord.*

JB. *I take it, then, that you share Brouwer's dim view of Cantor's set theory.*

LW. *Yes, but my objections to it are different from Brouwer's. Like Kant, Brouwer thinks that mathematics must be grounded in intuition, in particular, the intuition of time passing. The passage of time can never result in a completed infinity, so it was natural for Brouwer to exclude the concept of the completed infinite from mathematics. Now the idea of completed infinity is central to Cantor's set theory, so of course Brouwer became very critical of it. But, while their accounts of mathematics are radically different, they were both convinced that mathematics has a* content *deriving from outside the practice of mathematics itself. Cantor seems to have followed Plato in believing that concepts are latent in the mind and are "awakened" and brought to consciousness through some kind of natural process. These "awakened" concepts then become recognized as being elements of objective reality. In particular Cantor held this to be the case for mathematical concepts, including those central to his paradise, the completed infinite, transfinite numbers, and the like. This seems to me the sheerest metaphysical speculation. Brouwer surely recognized this. But in claiming that mathematics is a kind of distillation of temporal intuition and the activity of consciousness, Brouwer too makes the mistake of thinking that mathematics has a disguised content, that it is something other than what it actually is. I think that mathematics is* *a refined system of rules governing the manipulation of symbols which can be ascribed practical, useful meanings - as with numbers. The essential thing is that the rules of pure mathematics must be formulated in such a way that they apply directly to the symbols themselves, quite independently of whatever meaning is ascribed to the symbols. In this sense mathematics is pure syntax. And the mathematician's activity may be described as "playing a game" in that he is acting in accordance with certain rules.*

JB. *I see. And I guess that the fact the core rules of mathematics, for example the laws of arithmetic have remained unchanged for thousands of years is a natural source of the objectivity with which mathematics is usually credited. Take, for example, the question of whether there is an odd perfect number - not a burning mathematical issue but still unresolved, - unfinished mathematical business, so to speak. The problem - can an odd number be the sum of its divisors? - was known to the Pythagoreans around 550 B.C. (or so we are led to believe!). 2500 years later, the objective character of the problem is attested to by the confidence with which it usually claimed that it has - as a mathematical question- exactly the same meaning today as it did 2500 years ago. Thu,s what might be called the synchronicity of mathematics, its graspability by its adepts independently of its historical origins, is transformed into objectivity, or at least a stability over time. I think that an unsolved mathematical problem like the existence of odd perfect numbers can be likened to a lock constructed by a skilled locksmith, who has failed to provide a key for opening it. Once such a lock has been constructed, it becomes an objective matter as to whether a particular key opens it, and also whether such a key can actually be fashioned. Analogously, once a mathematical proposition has been formulated, it becomes an objective matter as to whether a proposed proof is actually a proof.*

LW. *I have had similar thoughts concerning philosophical problems. They can be compared to locks on safes, which can be opened by dialing a certain word or number, so that no force can open the door until just this word or number has been hit upon, and once it is hit upon any child can open it. Philosophy is like trying to open a safe with a combination lock: each little adjustment of the dials seems to achieve nothing, only when everything is in place does the door open. Of course, mathematics differs from philosophy in that once a problem is solved, understanding the solution is often beyond the average child, and requires the training of a mathematician.*

JB. *Fortunately, this not true of an art such as painting or music. To enjoy these it is not necessary to be trained as an artist or musician oneself.*

LW. *No, but full appreciation and enjoyment of an art, music especially, does require some awareness of how a work is put together. Are you interested in music? It's a great passion of mine.*

JB. *Yes, at one time I aspired to be a composer, and I went to Vienna to study music there.*

LW. *And who was your instructor?*

JB. *His name is Arnold Schoenberg.*

LW. *I hope you will not think me rude if I say I'm sorry to hear* that. *It seems to me that what Schoenberg and his colleagues call "the emancipation of the dissonance" and ""serial composition" is just vacuous posturing, the latest manifestation of the dccline music, and culture generally, hvve undergone since Brahms's time.*

JB. *I cannot agree with you there. Schoenberg was a curious mixture of revolutionary and conservative who believed that in introducing his revolutionary methods of composition he was actually combatting cultural decline. When he invented serialiasm, for example, he said, "I have made a discovery which will ensure the supremacy of German music for the next hundred years."   You may regard Schoenberg's declaration of "the emancipation of the dissonance" as vacuous posturing, but he felt that, as a composer, the only way to prevent Western classical music from degenerating into what he called* kitsch *was to enlarge the musical palette beyond the conventional use of the diatonic scale with its harmonic triads to the full use of the chromatic scale, which after all was already implicit in Bach's time with the introduction of equal temperament tuning of instruments. But his revolutionary fervour gave way to the uncomfortable realization that in abandoning tonality he had opened the door to musical anarchy. In Schoenberg's eyes the anarchy unleashed by atonalism had nothing directly to do with the corresponding loss of the euphony of diatonic composition – the loss, one might say, of sensible beauty. After all, there was nothing preventing an atonal composer from observing occasional adherence to tonal compositional practice – indeed the majority of atonal composers did exactly that. As Schoenberg himself is supposed to have said, "there is still plenty of good music to be written in C major". What concerned Schoenberg was the loss in the transition to atonality of the organizational principles governing tonal composition: major-minor triads, sonata form, and the like. In other words, he lamented the loss, not of sensible beauty but of what might be called intelligible beauty. The conservative in him thus came to seek new organizing principles for musical composition, once again, as with the Pythagoreans, based on mathematics. It was Schoenberg's conviction that order must be reimposed on chaos that led him to the invention of serialism, which, far from being "vacuous posturing", is an attempt to create a new aesthetic.   But this time, the Western compositional system, itself the result of piecemeal historical additions to the original Pythagorean insight that euphony as perceived by the human ear is built on simple arithmetical relations, was to be replaced by an organized structure built from a single universal musico-mathematical entity – the chromatic scale.*

LW. *Your explanation of Schoenberg's music has opened my eyes, but not my ears. Nevertheless you have convinced me of the depth of his character.*

JB. *He was a man much like yourself, as I have learned from this evening's conversation. Certainly you both have very strong convictions . I recall that as Schoenberg's student my admiration for him was tempered by my feeling that he lacked sympathy to the idea expressed in Terence's dictum* Homo sum: humani nihil a me alienum puto. *I hope you'll not be offended if I say that you remind me of him in that respect.*

LW. *Well, it would be silly to deny the truth, as it applies to me, of the first part of Terence's epigram. Of course, like him, I am a human being. But I deviate from the rest of his declaration. I often feel alienated from my fellow human beings, and this is only possible because* I myself *am a human being. It would be absurd if I felt alienated from a lion, say. I might fear or admire it, but it simply makes no sense to claim that I feel alienated from it, even though the words are easily uttered. And whatever the lion might feel about all this, if it could talk, we could not understand it. This is an excellent example of something showable, but not sayable. In attacking its prey the lion clearly shows what we call ferocity, when confined in a cage what we call frustration, and when wounded what we call pain. But if it could talk we would still not know whether its utterances in each situation bore any connection with its experience.*

JB. *Yes I agree, but still we might be very interested in understanding it, because then we would gain some inking of whaat it is like to be a lion.*

LW. *Yes, that's true. As a human being I already know what it is like to be human,so I'd be much more interested in finding out what it is like to be a lion. But I don't think tkat, even if we could understand the lion's language, we would have any better insight into what it is like to be that lion, any more than on hearing another person speak one's shared language gives one any kind of direct insight in what it is like actually* to be *that other person.*

JB. *May I take it, then, you regard the subjectivity of individual consciousness as essentially incommunicable? That, as Coleridge might have said "*the I-ness of I is known only to me"?

LW. *Yes, and I'd go further, the word "subjectivity" is sayable, but* subjectivity *is showable only to the individual subject irself. I rarely use the word. Subjectivity must be passed over in silence. Of course, in any case lions can't pronounce the word, as far as we know. But I see our fellow guests ricsing, so dinner is over. I've enjoyed our conversation. I often find it difficult to express myself, but you've succeedded in loosening my tongue, or is it the wine?*

JB. *I also enjoyed our discussion. I would never have guessed that you had any difficulty in expressing yourself.*

Part III

Miscellanea

Since his first meeting with the Time Traveller in London Bennhall had become fascinated with the idea of time travel, and he had started to think about how the possibility of travelling in time might affect the nature of reality. He began to set down his thoughts on the matter in a notebook. Here are a few entries.

My current reflections began with the mundane thought that each object in the universe is a "chrononaut" in the sense of being pulled forward, along with everything else, into the Future. Thus if one thinks of the objective passage of time from Past to Future as a river flowing in a particular direction, then individual objects can be likened to pieces of wood floating on the river's surface, carried along with its flow. The river's banks may then be thought of as Objective Reality, comprising all the events that occur at any time and in any place. In this conception, Objective Reality is itself timeless, and there is no objective distinction between Past and Future. Thus the Future "already exists" - is, one might say, "predetermined", or "baked in".

But how is one to treat the Present in this fluvial representation of the Real? Clearly the Present is the moving trace of the river's flow. Does this flow, this moving Present, exist objectively? And is the very idea of the flow of time compatible with the claim that the Future is not just predetermined, but in some sense already exists, exists *now?* Many have believed that the moving Present, the flow of time from Past to Future, does have objective existence: Newton, for example, did, and his teacher, Isaac Barrow, famously wrote:

Whether things move or are still, whether we sleep or wake, Time pursues the even tenor of its way.

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It has just struck me that if Objective Reality is truly timeless and both Past and Future are fixed, so speak, “in advance” , then time-travel into both Past and Future is already ”baked into” Objective Reality. It is commonly agreed that the Past cannot be changed. Everybody knows Omar Khayyam’s example, famous line

The Moving Finger writes; and, having written, moves on.

But if the Future is predetermined, it cannot be changed any more than can the Past, thus:

The Moving Finger does not write; it traces a text already written.

Thus on this account, if I use a time machine to travel 15 minutes into the future and this was achieved instantaneously, according to my own watch, on the objective level this would register as a “baked in” discontinuity in Objective Reality in that I would suddenly cease to exist for 15 minutes, at the end of which I would equally suddenly rematerialize. But continuity would be maintained in my own consciousness . On the other hand if I were simply to “go with the temporal flow” and sit out the fifteen minutes, then continuity would be maintained on both subjective and objective levels.

Yet, even if the future is fixed in advance, an objective, directed temporal “flow” from Past to Future could still exist, in the same way a river bed determines in advance the shape and direction of the flow of the water or a complex of railway tracks determines the paths of the trains traversing it. But if neither this flow nor the Present is “objective” then both must be subjective phenomena, and, as such, accessible only to a consciousness. In that case, one of the functions of consciousness must be the creation in subjectivity of the flow of time and the Present. In a sense, the Present constitutes all of subjective time, since it is the nature of thoughts to be displayed to consciousness only in the Present. Even memories of the Past and anticipations of the Future are summoned up in the Present.

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Any conscious being is aware of the dual nature of Time. the “subjective” Time of its own consciousness and the “objective” Time of the external world as manifested to it through the succession of actual events. If such a being were to journey into the Past, then this temporal duality would take a truly radical turn. In undertaking a journey into the Past , the time traveller’s subjective time, while still of course directed into the future from its own, subjective point of view, has become reversed from the standpoint of the external world and its objective time. Yet, if we' suppose the time-traveller's trip into the past not to be instantaneous, during its trip time will elapse for it in the usual way and will still be split into subjective and objective components. While the subjective component is simply a continuation of the subjective temporal flow of the time-traveller's consciousness before embarking on its journey into the past, the nature of the objective temporal component is less clear. In travelling into the past, the time-traveller's objective time has in some sense "deviated" from the objective time of the rest of the objects in the universe which are still proceeding into their (supposed common) future. It follows that the objective temporal component of the time-traveller's experience while undergoing its journey must have a local nature: it is a feature of the time-traveller's experience which is not shared by the rest of the objects in the universe travelling into their common Future. But, despite its locality, the objectivity of that component of the time-traveller's experience is indisputable since, for example, during the trip its body ages and its chronometer advances.

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An old man may in principle step into a time machine and revisit the past, even his past, but when he steps out of the machine he is no younger himself than when he entered it. In that respect there is no essential difference between the "objective" temporal journey and from the summoning up of the past through memory, or the contemplation of photographs of one's younger self. Gazing at a photograph of myself taken in my twenties, at a party, say, I might wish to be again the young man in the photograph. Now by this I surely do not mean that my present older self should employ a time machine to travel into the past and crash the party. (There, of course, I would have the curious experience of encountering my younger self, with the attendant problems of self-duplication.) In fact the wish to be young again could reasonably be taken to have two meanings. One is easily understood. It is the purely physical desire that one's body (and, perhaps, one's emotional perspective) should be restored to the state it was when one was younger. A possible way of achieving this is for my present older consciousness to be projected onto, or injected into, my past self in such a way as to share the mental space occupied by my past self. If both consciousnesses – past and future - were aware of the presence of the other, the principle of the unity of consciousness would be violated, possibly leading to joint mental breakdown. Another possibility is that the older self gains access to the contents of the consciousness of the younger self while the latter remains unaware of the presence of the older consciousness.

The wish to be young again could also be taken in the literal sense that my present older self should somehow come to coincide in the present with my younger self. This could be achieved by having both objective and subjective time run backwards in such a way as to induce my present self to metamorphose into my younger self. In classical mechanics this could, in principle, be done by reversing the direction of motion of all the particles in the universe. The result can be compared with a film run backwar*ds.* A good illustration here is Jean Cocteau's film *Orphée.* Cocteau shows his characters passing into the world of death by stepping through mirrors, and when he wants a character to spring back to life, he simply runs the film backwards. A memorable line from the film, bitter in its truth, is: Mirrors are the doors through which death comes and goes. Look at yourself in a mirror all your life and you'll see death do its work. Suppose that Cocteau had actually filmed someone staring into a mirror for his whole life and then reversed the film. One would see the person initially as old, and then, through time-reversal, slowly undergo a a transformation into his younger self. This an objective, observable effect. If sound recording had been used, the person's utterances would be reversed, and so become unintelligible as actual speech, but they would still be objective as sounds. Objectively, then, time-reversal seems clear enough. But subjectively, the effect of time-reversal is most unclear. What would it mean for the person's consciousness to "run backwards"? Of course, this question does not arise in the case at hand because the film only provides a record of the appearance of the person's body and sound of his speech as it is varies in objective time. No recording has been made of the flow of thoughts in the person's consciousness as it varies in (that person's) subjective time. But even if that subjective flow could be "recorded" in some way, it is not at all clear what temporal reversal would mean from a subjective point of view. The closest one can come to this seems to be the effort one sometimes makes in memory to return to a previous point in the flow of one's thoughts, for example when one attempts to trace the source of a thought by summoning up previous mental associations. But of course this is not done by "reversing" the flow of consciousness. It is not the subjective temporal equivalent of tying a knot and then untying it. In attempting to return through memory to a previous point in one's conscious life the flow of consciousness still proceeds "forward". What memory presents is a discrete chain of previous associations, from which the continuous flow accompanying their actual emergence in one's mind has been discarded. Nevertheless, the consciousness in which the memory operates in the "now" is always continuous, even if its object of attention, in this case a sequence of previous associations, is discrete.

The fact is that if one's wish to be young again requires that objective and subjective time run backwards in such a way as to induce one's present older self to metamorphose into one's younger self, then that would also seemingly require the effacing of one's older self (and, perhaps, of everything contemporaneous with it.) But this would amount to truncating the universes' past at the time the photograph of one's younger self was taken; in other words, chopping off all the worldlines of objects (including, of course own's own body along with its consciousness) at that time. One can recall one's past self, and imagine returning to the state of that past self, at the same tine summoning up the universe in which that past self was immersed, thus effacing the Future. This effacement can be likened to the cutting back of a vine. At any given moment, the vine in its entirety displays all of its stages of growth up to that moment. When it is cut back, each of its tendrils (and so the whole vine, if the cutting is performed uniformly) is "returned" to a stage of growth corresponding to an earlier time.

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Now that I have become an active chrononaut, using the Liberator to visit past and future, I have come to think of events in time as akin to locations in space. Just as when one proposes a visit to the Statue of Liberty, say, one expects the Statue of Liberty to exist in a concrete sense independently of the time one chooses to visit it, so I now think of future events as existing in a similar concrete sense independently of when I, or any chrononaut might choose to experience them. In fact I now regard the Future as predetermined. But this does not decide the question of whether an objective flow of time exists. And, as I expected, becoming a chrononaut has not had the slightest effect on my own subjective time-consciousness.

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Towards the end of his life, Bennhall penned the following essay, which was eventually published as a chapter in the 60th birthday *Festschrift* of John Lane Bell, a minor logician and philosopher, whom Bennhall had met in London not long before his death, and on whom he had made an indelible impression.

Chapter 26
Inscrutable Harmonies: The Continuous and the Discrete as Reflected in the Playing of Jascha Heifetz and Glenn Gould

Joel Bennhall

We owe the Pythagoreans the revelation that the harmonies of music derive from number, that is, from the discrete. This must be seen as a triumph of inscrutability. Inscrutable indeed is the resulting subsumption of music within mathematics, a colourless, forbidding subject to most, indeed the polar opposite of music, whose gaudy, yet profound, epiphanies offer a striking contrast. The Pythagoreans are to be blamed for the fact that music came to find itself in the embrace of such unlikely bedfellows as arithmetic, geometry and astronomy, the other members of the mathematical *quadrivium*.

Still, this evidence might cause a Marxist to respond that, while the basis of musical *organization* is to be located in the discrete, its *means of production* originate in the realm of the continuous. For is not sound itself nothing more or less than a continuous vibratory excitation of the atmospheric envelope, whether induced by blowing, plucking, or striking a tensed string, beating a drum, blowing down a tube, or straining one's vocal cords? The Pythagorean discovery, at bottom, is an instance of *ex continuo discretum*.

Musical instruments may be classed as continuous or discrete according to the manner in which individual notes are sounded. Thus the voice, bowed string instruments, and slide trombones are naturally identified as continuous, while valved wind instruments such as the clarinet or oboe, plucked or struck string instruments such as the lute or dulcimer, and keyboard instruments such as the harpsichord or piano may be classified as discrete.

J. Bennhall (✉)
Musicologist and Composer

Editors' note: The musicologist and composer Joel Bennhall was briefly, when very young, a pupil of the Danish composer Dag Henrik Esrum-Hellerup. Later he studied with Schoenberg. His fascination with the relationship between music and mathematics led him to create topomusicological analysis, at one time a rival in musicological circles to Hans Keller's better known functional analysis. In his old age Bennhall met the man celebrated in this volume at a meeting of the London Melomaniacal Society, at which he read the present paper; sadly he was prevented from publishing it owing to his sudden demise (at the age of 98). The Editors are pleased to provide it with a suitable resting place.

D. DeVidi et al. (eds.), *Logic, Mathematics, Philosophy: Vintage Enthusiasms*, 467
The Western Ontario Series in Philosophy of Science 75,
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Accomplished vocalists and players of continuous instruments have considerable freedom both in determining the quality of individual notes and in the shaping of the "line" engendered by the succession of notes. This freedom is manifested above all in the case of the violin. In the violin the continuous and the discrete are truly united. For while the violinist's bow is the source *par excellence* of continuous sound, of a variable intensity controlled by subtle alterations in pressure of the fingers on the strings, in the hands of a virtuoso that very bow is also employed to spectacular effect in engendering discreteness: witness, for example, *spiccato*, *staccato* and *col legno* bowing.

With the violinist's left hand the order of continuity and discreteness is reversed, since the violinist's digits are employed in the first instance to produce separate discrete notes through "stopping" the strings. But just as the bow can generate discreteness, so can the left hand generate continuity, e.g. through *vibrato*, the continuous minute oscillation of pitch of a single note[1]; the *portamento*, the subtle continuous movement from one note to another by gently gliding the finger along the string; and the *shift*, the violinist's equivalent of the mathematician's continuous change of coordinate system.

While the discrete instruments lack these refinements, they have one great advantage over their continuous counterparts, namely, their capacity to support *simultaneity*. A mere tyro on the guitar plays multiply voiced chords as a matter of course, while even a competent violinist may have difficulty in playing double-stops in tune. With the keyboard instruments this natural capacity to engender simultaneity has achieved its highest development in the polyphonic structures created by the composers of the Baroque period, and above all by J.S. Bach. Bach raises keyboard polyphony to undreamt-of heights, and certainly to a level far surpassing that achievable on any single stringed instrument. Even that most elaborate four-part fugue in the C major solo sonata cannot compare in complexity with the cyclopean edifice of the Art of Fugue!

Now let us turn to consider two modern masters of their respective instruments—Jascha Heifetz, the violinist whose technical command of the instrument is widely regarded as supreme—and Glenn Gould, the wizard of piano polyphony. The one, the master of the continuous, the other, the master of the discrete.

Jascha Heifetz—a name with which to conjure. For the present writer the name evokes a cluster of associations, each of which involves the continuous in one way or another. One recalls for example the famous exchange at the young Heifetz's New York debut between two members of an audience packed with musicians eager to hear the new *Wunderkind*—Mischa Elman, great violinist, and Leopold Godowsky, equally great pianist. To Elman's ingenuous observation, "It's hot in here, isn't it?", no wittier response is conceivable than Godowsky's "Yes, but not for pianists!" In this instance the discrete could afford to smile at the embarrassment of

[1] The *trill*—the rapid alternation of the main note with that a tone or semitone above it—is of course a discrete effect.

the continuous. (And yet it must be recalled that pianist-composers such as Liszt and Chopin were strongly influenced by the virtuosity of Paganini.)

Consider also Heifetz's celebrated definition of a Russian: one Russian—an anarchist; two Russians—a game of chess; three Russians—a revolution; four Russians—the Budapest String Quartet. Here one sees a striking progression from the pure discreteness of the unit to the continuity of stringed instruments.

Even Virgil Thomson's nastily dismissive description of Heifetz's repertoire as "silk underwear music" brings the continuous to mind.

But in truth Heifetz *was* the supreme master of silkiness, indeed of that ultimate form of continuity that mathematicians call *smoothness*. This quality is best heard in his recordings of the 1930s and 1940s, most arrestingly in encore pieces and lightweight concertos such as those of Korngold and Gruenberg. In these latter works the silky smoothness of Heifetz's tone—endlessly imitated but never duplicated by generations of violinists under his spell—almost surpasses belief.

Mathematicians have introduced the concept of a smooth topos, a mathematical "world" in which all correlations are arbitrarily many times differentiable, there are no jagged edges and in which, in Leibniz's words, *natura non facit saltus*. There is no question that Heifetz would be the canonical fiddler in such a world.

But Heifetz was also a master of the discrete effects achievable on the violin; above all he could play detached notes on his preferred Guarnerius with blinding celerity. A remarkable example of his facility in this respect is provided by his recording of the Sinding *Suite*, in which the first, *presto* movement is despatched with truly hair-raising speed and accuracy. There could not be a more striking contrast between this glittering flurry of notes and the smooth, yet sweetly earnest and heartfelt manner in which Heifetz delivers the second, *adagio* movement.

The present writer did not have many opportunities to see Heifetz on the concert platform, but they remain treasured experiences. Especially memorable was the recital at the Royal Festival Hall in the 1950s at which Heifetz was due to begin with the Vitali *Chaconne*. This piece, familiar to all violinists, begins with a G minor chord, so when Heifetz picked up his violin and struck a G major chord the ranks of violinists occupying the first few rows of seats fell back in shock. Heifetz nonchalantly went on to play the English national anthem in G major.

Fortunately there are in existence a handful of filmed performances of Heifetz. One of the most remarkable of these is the rendition of that famed encore piece the *Hora Staccato*, transcribed by Heifetz from a Rumanian original. The writer once had the experience of hearing this exacting morsel played by a gypsy fiddler in an Amsterdam restaurant. While adequate, the performance could not compare with that of Heifetz, who contrives to make the staccato effect *sound* discrete but *appear* to the eye as continuous.

Now let us turn to Glenn Gould. While he was of course a master of the keyboard, with an unexampled command of polyphonic technique, one suspects that he may have envied the string player and the singer their immediate contact with the continuous. Grounds for this surmise are provided by his admitted inability to suppress the vocalise which invariably accompanied his piano playing, and which was such a source of vexation to critics and listeners alike.

On the other hand, for Gould, in the final analysis, polyphony was all, and polyphony, on the piano at least, is achieved by the systematic exploitation of discreteness. So it is reasonable to suppose that at some point Gould made the conscious decision to celebrate the discreteness of the piano, to avoid the mimicking of continuity by what he saw as contrived and hackneyed effects such as overpedalling and the gratuitous use of legato. Thus he strove for a *secco, detaché* sound, with each individual note rejoicing in its separateness. This approach is extraordinarily effective in Baroque works; and also with the twentieth century composers Gould most admired: Schoenberg, Webern, Krenek,Hindemith. In the present writer's opinion, the approach is also effective in Beethoven, especially in the early works of that master. With Mozart, however, the result is, to this writer's ears at least, nothing short of disastrous—but this was, of course, exactly what Gould, who disliked Mozart's music (with the conspicuous exception of the early sonatas K. 279–284 and the Fantasy and Fugue K. 394) was trying to achieve. Here Gould carried discrete deconstruction to the point of destruction.

It is to Gould's transcendent performances of works by composers he esteemed that one turns again and again. And above all, of course, to the compositions of J.S. Bach. Although Gould was most famed for his recordings of the *Goldberg Variations*—a fame that led this work to be identified by his fans as the "Gouldberg" Variations—the composition of Bach's he revered above all others was the *Art of Fugue*. And yet Gould produced no complete recording of this supreme, but alas, unfinished, masterpiece of the polyphonist's art. For this writer the most exciting rendition of any part of this work is Gould's 1967 Canadian radio broadcast of Contrapuncti IX, XI and XIII. Here Gould's playing achieves what can only be described as an ecstatic seamlessness fusing discreteness and continuity in an almost Hegelian *Aufhebung*.

Finally, we must consider the question of how Heifetz and Gould would have sounded had they played together. Would these supreme exponents of continuity and discreteness have achieved a harmonious union?

The vast majority of Heifetz's duo recordings were made with contract pianists—able, but somewhat colourless. An exception is the magnificent recording of Brahms' op. 108 sonata Heifetz made in the 1950s with the brilliant American pianist William Kapell (who died tragically young). Here the power of the pianist's playing comes close to matching Heifetz's, driving the latter to peaks even he did not always attain with his usual accompanists.

As for Gould, he made only a handful of recordings with violinists. One recalls the curious *rencontre* with Yehudi Menuhin during which Gould persuaded the violinist to play the Schoenberg *Fantasy* op. 47, a work to which, like all of Schoenberg's output, Gould was partial, but which Menuhin later said he found totally incomprehensible. In this connection it is pertinent to recall Heifetz's similar antipathy to Schoenberg's *oeuvre*. Heifetz actually commissioned Schoenberg's Violin Concerto op. 36 but on seeing the score instantly rejected it, giving the scarcely credible excuse that to play it would require him to grow a sixth finger. "I can wait," Schoenberg is reputed to have replied.

Gould did record the Bach violin and keyboard sonatas with Jaime Laredo, a good violinist, but who takes a back seat to Gould. The one string player who really stood up to Gould in duet performance was the cellist Leonard Rose, who, in their recording of Bach's sonatas for cello and keyboard gives a robust performance fully matching Gould's powerful rendition.

The upshot is that we can only imagine the sound of a Heifetz/Gould recital. One's musical tongue waters at the idea of recordings by these two masters of the Bach, Beethoven, or Brahms sonatas. The nearest approach we can make to this ideal is to listen to the pair of Bach violin concertos (in E major and A minor) and their keyboard transcriptions (in D major and G minor) as recorded, respectively, by Heifetz and Gould. It is a rare treat to hear Bach's sublime lines played first continuously, and then with discrete elaboration.

If only Heifetz and Gould had collaborated! That would have been the ultimate synthesis of the continuous and the discrete.