Hermann Weyl: Mathematician-Philosopher

MATHEMATICS AND PHILOSOPHY ARE CLOSELY LINKED, and several great mathematicians who were at the same time great philosophers come to mind—Pythagoras, Descartes and Leibniz, for instance. One great mathematician of the modern era in whose thinking philosophy played a major role was *Hermann Weyl* (1885–1955), whose work encompassed analysis, number theory, topology, differential geometry, relativity theory, quantum mechanics, and mathematical logic. His many writings are informed by a vast erudition, an acute philosophical awareness, and even, on occasion, a certain playfulness. No matter what the subject may be—mathematics, physics, philosophy—Weyl's writing fascinates both by the depth of insight it reveals and by its startling departures from academic convention. Who else would have the daring to liken (as he does in the discussion of Space and Time in his *Philosophy of Mathematics and Natural Science*), a coordinate system to "the residue of the annihilation of the ego"[1]? Or then (somewhat further on in the same discussion) to express the belief in the impossibility of a completely objective account of individual consciousness by the assertion "…it is shattered by Judas' desperate outcry, 'Why did *I* have to be Judas?'"[2].

Philosophy, more particularly idealist philosophy, played an important role in Weyl's thought right from the beginning. In his short philosophical autobiography of 1954, *Insight and Reflection,* Weyl tells of the impact made on him as a schoolboy by a commentary on Kant's "Critique of Pure Reason." He was especially taken with Kant's doctrine that space and time are not inherent in the objects of the world, existing as such and independently of our awareness, but are, rather, *conceptual forms* or *intuitions* based in our intellects. And he goes on to quote Fichte—whose language he describes as "always a bit eccentric", but whose philosophic idealism was nevertheless to exert a considerable intellectual influence on him—

> *Transparent penetrable space, the purest image of my knowing, cannot be inspected but must be seen intuitively, and within it my inspecting itself is so seen. The light is not outside of me, but rather in me.*[3]

Although Weyl came to abandon the greater part of Kant's doctrines, he cleaved always to the idea of the primacy of intuition that he had first learned from Kant.

Intuition was central to Weyl's epistemology. Towards the end of his *Address on the Unity of Knowledge*, delivered at the 1954 Columbia University bicentennial celebrations, Weyl enumerates what he considers to be the essential constituents of knowledge. At the top of his list[4] comes

> *…intuition, mind's ordinary act of seeing what is given to it.*[5]

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[1] Weyl, *Philosophy*, 123. This metaphor seems first to have appeared in *The Continuum*, where Weyl asserts on p. 94:

> The coordinate system is the unavoidable residue of the eradication of the ego in that geometrico-physical world which reason sifts from the given using "objectivity" as its standard—a final scanty token in this objective sphere that existence is only given and *can* only be given as the intentional content of the processes of consciousness of a pure, sense-giving ego.

[2] *Ibid.*, 124–125.

[3] Quoted in Weyl [1969], 282.

[4] The others, in order, are: *understanding and expression; thinking the possible;* and finally, in science, *the construction of symbols or measuring devices.*

[5] Weyl [1954], 629.

In particular Weyl held to the view that intuition, or *insight*—rather than *proof*—furnishes the ultimate foundation of *mathematical* knowledge. Thus in his *Das Kontinuum* of 1918 he says:

> *In the Preface to Dedekind (1888) we read that "In science, whatever is provable must not be believed without proof." This remark is certainly characteristic of the way most mathematicians think. Nevertheless, it is a preposterous principle. As if such an indirect concatenation of grounds, call it a proof though we may, can awaken any "belief" apart from assuring ourselves through immediate insight that each individual step is correct. In all cases, this process of confirmation—and not the proof—remains the ultimate source from which knowledge derives its authority; it is the "experience of truth".*[6]

Weyl's initial Kantianism crumbled soon after he entered Göttingen University in 1904. There he read Hilbert's Foundations of Geometry, a tour-de-force of the axiomatic method, in comparison to which Kant's "bondage to Euclidean geometry" now seemed seemed to Weyl naïve.After this philosophical debâcle Weyl seems to have lapsed for a while into an indifferent positivism. In 1912–13 his interest in philosophy was rekindled by his coming to learn of Husserl's phenomenology, to which he had been introduced by his wife, a student of Husserl's. It was also at about this time that Fichtean metaphysical idealism came to "capture his imagination."

Athirst for philosophy, Weyl cannot have been reluctant to abandon the aridities of positivism for the potential springs of phenomenology, whose goal he describes as

> *... to capture phenomena in their essential being—purely as they yield themselves apart from all genetical and other theories in the encounter with our consciousness.*[7]

The principal tenet of phenomenology is that the only things which are directly given to us, that we can know completely, are objects of consciousness. It is these with which philosophy, and all knowledge, must begin. This is acknowledged by Weyl in the introduction to *Space-Time-Matter* (1919), his famous book on the theory of relativity. Modestly describing his remarks as "a few reflections of a philosophical character," he observes that the objective world "constructed" by mathematical physics cannot of necessity coincide with the subjective world of qualities given through perception:

> *Expressed as a general principle, this means that the real world, and every one of its constituents, are, and can only be, given as intentional objects of acts of consciousness. The immediate data which I receive are the experiences of consciousness in just the form in which I receive them ... we may say that in a sensation an object, for example, is actually physically present for me—to whom that sensation relates—in a manner known to everyone, yet, since it is characteristic, it cannot be described more fully. (Weyl, Space-Time-Matter, 4)*

His phenomenological orientation is proclaimed still more emphatically when he goes on to say:

> *...the datum of consciousness is the starting point at which we must place ourselves if we are to understand the absolute meaning of, as well as the right to, the supposition of reality ... "Pure consciousness" is the seat of what is philosophically a priori. (ibid., 5)*

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[6] Weyl [1987], 119.

[7] *Ibid.,* 288.

Later, he asserts that

> *Time is the ... form of the stream of consciousness ...and space... the form of external material reality. (ibid.,5)*

And in a memorable passage he describes how these two opposed facets of existence ineluctably come to be fused, so leading to the amalgamation of time and space, of which the theory of relativity is (thus far) the deepest expression:

> *... if the worlds of consciousness and of transcendental reality were totally different from one another, or, rather, if only the passive act of perception bridged the gulf between them, the state of affairs would remain as I have represented it, namely, on the one hand a consciousness rolling on in the form of a lasting present, yet spaceless; on the other, a reality spatially extended, yet timeless, of which the former contains but a varying appearance. Antecedent to all perception there is in us the experience of effort and opposition, of being active and being passive. For a person leading a natural life of activity, perception serves above all to place clearly before his consciousness the definite point of the action he wills, and the source of the opposition to it. As the doer and endurer of actions I become a single individual with a psychical reality attached to a body which has its place in space among the material things of the external world, and by which I am in communication with other similar individuals. Consciousness, without surrendering its immanence, becomes a part of reality, becomes this particular person, myself, who was born and will die. Moreover, as a result of this, consciousness spreads out its web, in the form of time, over reality. Change, motion, elapse of time, coming and ceasing to be, exist in time itself; just as my will acts on the external world through and beyond my body as a motive power, so the external world is in its turn active. Its phenomena are related throughout by a causal connection. In fact, physics shows that cosmic time and physical time cannot be disassociated from one another. The new solution of the problem of amalgamating space and time offered by the theory of relativity brings with it a deeper insight into the harmony of action in the world. (ibid., 6)*

Thus, for Weyl, the duality between mind and material reality leads to a unity between space and time.

In *The Open World* (1932), Weyl provides an eloquent formulation of his philosophical outlook, which quickly moves beyond its initial echoes of Schopenhauer:

> *The beginning of all philosophical thought is the realization that the perceptual world is but an image, a vision, a phenomenon of our consciousness; our consciousness does not directly grasp a transcendental real world which is as it appears. The tension between subject and object is no doubt reflected in our conscious acts, for example, in sense perceptions. Nevertheless, from the purely epistemological point of view, no objection can be made to a phenomenalism which would like to limit science to the description of what is "immediately given to consciousness". The postulation of the real ego, of the thou and of the world, is a metaphysical matter, not judgment, but an act of acknowledgment and belief. But this belief is after all the soul of all knowledge.* [8]

Here the *transcendental real world* is the "objective" realm beyond immediate consciousness, the world with which theoretical physics, for example, is concerned. Since this domain is inscrutable to intuition, it can only be charted *indirectly,* through

[8] Weyl [1932], 26-27.

the medium of what Weyl calls *symbolic construction*, or *theoretical creation.* He concludes his *Current Epistemological Situation in Mathematics* of 1925 with a passage in which knowledge obtained in this indirect fashion is contrasted with that given purely in intuition:

> *Theories permit consciousness to "jump over its own shadow", to leave behind the given, to represent the transcendent, yet, as is self-evident, only in symbols. It never leads, I believe, to a final result, like phenomenal knowledge, which, although subject to human error, is nevertheless by its nature immutable.*[9]

Although Weyl held that the roots of mathematics lay in the intuitively given as opposed to the transcendent, he recognized at the same time that it would be unreasonable to require all mathematical knowledge to possess intuitive immediacy. In *Das Kontinuum*, for example, he says:

> *The states of affairs with which mathematics deals are, apart from the very simplest ones, so complicated that it is practically impossible to bring them into full givenness in consciousness and in this way to grasp them completely.*[10]

But Weyl did not think that this fact furnished justification for extending the bounds of mathematics to embrace notions which cannot be given fully in intuition *even in principle* (e.g., the actual infinite). He held, rather, that this extension of mathematics into the transcendent is necessitated by the fact that mathematics plays an indispensable role in the physical sciences, in which intuitive evidence is *necessarily* transcended. As he says in *The Open World*::

> *... if mathematics is taken by itself, one should restrict oneself with Brouwer to the intuitively cognizable truths ... nothing compels us to go farther. But in the natural sciences we are in contact with a sphere which is impervious to intuitive evidence; here cognition necessarily becomes symbolical construction. Hence we need no longer demand that when mathematics is taken into the process of theoretical construction in physics it should be possible to set apart the mathematical element as a special domain in which all judgements are intuitively certain; from this higher standpoint which makes the whole of science appear as one unit, I consider Hilbert to be right.* [11]

In *Consistency in Mathematics* (1929), Weyl characterized the mathematical method as

> *the* a priori *construction of the possible in opposition to the* a posteriori *description of what is actually given.*[12]

The problem of identifying the limits on constructing "the possible" in this sense occupied Weyl a great deal. He was particularly concerned with the concept of the mathematical *infinite*, which he believed to elude "construction" in the idealized sense of set theory. Again to quote a passage from *Das Kontinuum:*

> *No one can describe an infinite set other than by indicating properties characteristic of the elements of the set. ... The notion that a set is a*

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[9] Weyl [1998a], 140.
[10] Weyl [1987], 17.
[11] Weyl [1932], 82.
[12] Weyl [1929], 249.

> *"gathering" brought together by infinitely many individual arbitrary acts of selection, assembled and then surveyed as a whole by consciousness, is nonsensical; "inexhaustibility" is essential to the infinite.*[13]

But the necessity of injecting mathematics into external reality compels it to embody a conception of the actual infinite, as Weyl attests towards the end of *The Open World:*

> *The infinite is accessible to the mind intuitively in the form of a field of possibilities open to infinity, analogous to the sequence of numbers which can be continued indefinitely, but the completed, the actual infinite as a closed realm of actual existence is forever beyond its reach. Yet the demand for totality and the metaphysical belief in reality inevitably compel the mind to represent the infinite as closed being by symbolical construction.*[14]

Another mathematical "possible" to which Weyl gave a great deal of thought is the idea of the *continuum.* During the period 1918–1921 he wrestled with the problem of providing it with an exact mathematical formulation free of the taint of the actual infinite. As Weyl saw it, there is an unbridgeable gap between intuitively given continua (e.g. those of space, time and motion) on the one hand, and the "discrete" exact concepts of mathematics (e.g. that of real number) on the other. For Weyl the presence of this split meant that the construction of the mathematical continuum could not simply be read off" from intuition. Rather, he believed at this time that the mathematical continuum must be treated as if it were an element of the transcendent realm, and so, in the end, justified in the same way as a physical theory. In Weyl's view, it was not enough that the mathematical theory be *consistent*; it must also be *reasonable.*

In his *Das Kontinuum* of 1918, Weyl's attempts to formulate a theory of the continuum which satisfies the first, and, as far as possible, the second, of these requirements. In the following passages from this work he acknowledges the difficulty of the task:

> *... the conceptual world of mathematics is so foreign to what the intuitive continuum presents to us that the demand for coincidence between the two must be dismissed as absurd.*[15]

> *... the continuity given to us immediately by intuition (in the flow of time and of motion) has yet to be grasped mathematically as a totality of discrete "stages" in accordance with that part of its content which can be conceptualized in an exact way.*[16]

> *Exact time- or space-points are not the ultimate, underlying atomic elements of the duration or extension given to us in experience. On the contrary, only reason, which thoroughly penetrates what is experientially given, is able to grasp these exact ideas. And only in the arithmetico-analytic concept of the real number*

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[13] Weyl [1987], 23.
[14] Weyl [1932], 83.
[15] Weyl [1987], 108.
[16] *Ibid.,* 24. In this connection it is of interest to note that Brentano (whose ideas also exerted an influence on Weyl), in his *On What is Continuous* of 1914, had drawn the similar conclusion that the continuum concept is derived from primitive sensible intuition and indeed that "all our sensible intuitions present us with that which is continuous." This led him to regard the constructions of the continuum of Dedekind, Cantor, and their successors as "fictions".

> *belonging to the purely formal sphere do these ideas crystallize into full definiteness.* [17]
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> *When our experience has turned into a real process in a real world and our phenomenal time has spread itself out over this world and assumed a cosmic dimension, we are not satisfied with replacing the continuum by the exact concept of the real number, in spite of the essential and undeniable inexactness arising from what is given.*[18]

However much he may have wished to do so, in *Das Kontinuum* Weyl did not aim to provide a mathematical formulation of the continuum as it is presented to intuition, which, as the quotations above show, he regarded as an impossibility (at that time at least). Rather, his goal was first to achieve *consistency* by putting the *arithmetical* notion of real number on a firm logical basis, and then to show that the resulting theory is *reasonable* by employing it as the foundation for a plausible account of continuous process in the objective physical world.[19]

As a practicing mathematician, Weyl had come to believe that, the work of Cauchy, Weierstrass, Dedekind and Cantor notwithstanding, mathematical analysis at the beginning of the 20th century would not bear logical scrutiny, for its essential concepts and procedures involved vicious circles to such an extent that, as he says, "every cell (so to speak) of this mighty organism is permeated by contradiction." In *Das Kontinuum* he tries to overcome this by providing analysis with a *predicative* formulation—not, as Russell and Whitehead had attempted, by introducing a hierarchy of logically ramified types, which Weyl seems to have regarded as too complicated—but rather by confining the comprehension principle to formulas whose bound variables range over just the initial given entities (numbers). Thus he restricts analysis to what can be done in terms of natural numbers with the aid of three basic logical operations, together with the operation of substitution and the process of "iteration", i.e., primitive recursion. Weyl recognized that the effect of this restriction would be to render unprovable many of the central results of classical analysis—e.g., Dirichlet's principle that any bounded set of real numbers has a least upper bound[20]—but he was prepared to accept this as part of the price that must be paid for the security of mathematics.

But by 1919 Weyl had come to repudiate atomistic theories of the continuum, including that of his own *Das Kontinuum* and now embraced Brouwer's radical account on the mathematical continuum. The latter's influence looms large in Weyl's next paper on the subject, *On the New Foundational Crisis of Mathematics,* written in 1920. Here Weyl identifies two distinct views of the continuum: "atomistic" or "discrete"; and "continuous". In the first of these the continuum is composed of individual real numbers which are well-defined and can be sharply distinguished. Weyl describes his earlier attempt at reconstructing analysis in *Das Kontinuum* as atomistic in this sense:

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[17] *Ibid.,* 94.

[18] *Ibid.,* 93.

[19] The connection between mathematics and physics was of course of paramount importance for Weyl. His seminal work on relativity theory, *Space-Time-Matter,* was published in the same year (1918) as *Das Kontinuum*; the two works show subtle affinities.

[20] In this connection it is of interest to note that on 9 February 1918 Weyl and George Pólya made a bet in Zürich in the presence of twelve witnesses (all of whom were mathematicians) that "within 20 years, Pólya, or a majority of leading mathematicians, will come to recognize the falsity of the least upper bound property." When the bet was eventually called, everyone—with the single exception of Gödel—agreed that Pólya had won.

> *Existential questions concerning real numbers only become meaningful if we analyze the concept of real number in this extensionally determining and delimiting manner. Through this conceptual restriction, an ensemble of individual points is, so to speak, picked out from the fluid paste of the continuum. The continuum is broken up into isolated elements, and the flowing-into-each other of its parts is replaced by certain conceptual relations between these elements, based on the "larger-smaller" relationship. This is why I speak of the* atomistic *conception of the continuum.*[21]

While intuitive considerations, together with Brouwer's influence, must certainly have fuelled Weyl's rejection of such theories, it also had a *logical* basis. For Weyl had come to regard as meaningless the formal procedure—employed in *Das Kontinuum*—of negating universal and existential statements concerning real numbers conceived as developing sequences or as sets of rationals. This had the effect of undermining the whole basis on which his theory had been erected, and at the same time rendered impossible the very formulation of a "law of excluded middle" for such statements. Thus Weyl found himself espousing a position[22] considerably more radical than that of Brouwer, for whom negations of quantified statements had a perfectly clear constructive meaning, under which the law of excluded middle is simply not generally affirmable.

Above and beyond the claims of logic, Weyl welcomed Brouwer's construction of the continuum by means of sequences generated by free acts of choice, thus identifying it as a "medium of free Becoming" which "does not dissolve into a set of real numbers as finished entities". Weyl felt that Brouwer, through his doctrine of Intuitionism[23], had come closer than anyone else to bridging that "unbridgeable chasm" between the intuitive and mathematical continua. In particular, he found compelling the fact that the Brouwerian continuum is not the union of two disjoint nonempty parts—that it is, in a word, *indecomposable.* "A genuine continuum," Weyl says, "cannot be divided into separate fragments." In later publications he expresses this more colourfully by quoting Anaxagoras to the effect that a continuum "defies the chopping off of its parts with a hatchet."

Weyl also agrees with Brouwer that all functions everywhere defined on a continuum are continuous, but here certain subtle differences of viewpoint emerge. Weyl contends that what mathematicians had taken to be discontinuous functions actually consist of several continuous functions defined on separated continua. (For example, the "discontinuous" function defined by $f(x) = 0$ for $x < 0$ and $f(x) = 1$ for $x \geq 0$ in fact consists of the two functions with constant values 0 and 1 respectively defined on the separated continua $\{x: x < 0\}$ and $\{x: x \geq 0\}$. The union of these two continua fails to be the whole of the real continuum because of the failure of the law of excluded middle: it is not the case that, for be any real number x, either $x < 0$ or $x \geq 0$.) Brouwer, on the other hand, had not dismissed the possibility that discontinuous functions could be defined on proper parts of a continuum, and still seems to have been searching for an appropriate way of formulating this idea.[24] In particular, at that time Brouwer would probably have been inclined to regard the above function f as a genuinely discontinuous

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[21] Weyl [1998], 91.

[22] Weyl's contention is strikingly similar to (and may have had an influence on) Hilbert's later assertion that "contentual" statements are, from the finitist standpoint, incapable of being negated. See, e.g., Hilbert [1926], 378.

[23] For my remarks on Weyl's relationship with Intuitionism I have drawn on the illuminating paper van Dalen [1995].

[24] Brouwer established the continuity of functions fully defined on a continuum in 1904, but did not publish a definitive account until 1927. In that account he also considers the possibility of partially defined functions.

function defined on a *proper part* of the real continuum. For Weyl, it seems to have been a self-evident fact that all functions defined on a continuum are continuous, but this is because Weyl confines attention to functions which turn out to be continuous by definition. Brouwer's concept of function is less restrictive than Weyl's and it is by no means immediately evident that such functions must always be continuous.

Weyl defined real functions as mappings correlating each interval in the choice sequence determining the argument with an interval in the choice sequence determining the value "interval by interval" as it were, the idea being that approximations to the input of the function should lead effectively to corresponding approximations to the input. Such functions are continuous by definition. Brouwer, on the other hand, considers real functions as correlating choice sequences with choice sequences, and the continuity of these is by no means obvious. The fact that Weyl refused to grant (free) choice sequences—whose identity is in no way predetermined—sufficient individuality to admit them as arguments of functions perhaps betokens a commitment to the conception of the continuum as a "medium of free Becoming" even deeper than that of Brouwer.

There thus being only minor differences between Weyl's and Brouwer's accounts of the continuum, Weyl accordingly abandoned his earlier attempt at the reconstruction of analysis and "joined Brouwer." At the same time, however, Weyl recognized that the resulting gain in intuitive clarity had been bought at a considerable price, as witnessed by his remark in the 1927 edition of *Philosophy of Mathematics and Natural Science:*

Mathematics with Brouwer gains its highest intuitive clarity. He succeeds in developing the beginnings of analysis in a natural manner, all the time preserving the contact with intuition much more closely than had been done before. It cannot be denied, however, that in advancing to higher and more general theories the inapplicability of the simple laws of classical logic eventually results in an almost unbearable awkwardness. And the mathematician watches with pain the greater part of his towering edifice which he believed to be built of concrete blocks dissolve into mist before his eyes.[25]

Although he later practiced intuitionistic mathematics very rarely, Weyl remained an admirer of intuitionism. And the "riddle of the continuum" retained its fascination for him: in one of his last papers, *Axiomatic and Constructive Procedures in Mathematics,* written in 1954, we find the observation that

> *... the constructive transition to the continuum of real numbers is a serious affair... and I am bold enough to say that not even to this day are the logical issues involved in that constructive concept completely clarified and settled.*[26]

It seems appropriate here to quote the passage, from a review paper of 1946, in which Weyl summarizes the effect that the problem of foundations had had on his own work:

This history should make one thing clear: we are less certain than ever work about the ultimate foundations of (logic and) mathematics; like everybody and everything in the world today, we have our "crisis". We have had it for nearly fifty years. Outwardly it does not seem to hamper our daily work, and yet I for one confess that it has had a considerable practical influence on my mathematical life: it directed my interests to fields I considered relatively "safe", and it has been a constant drain on my enthusiasm and determination with which I pursued my research work. The experience is probably shared by other mathematicians who are not indifferent to what their scientific endeavours mean

[25] Weyl [1963], 54.
[26] Weyl [1985], 17.

in the contexts of man's whole caring and knowing, suffering and creative existence in the world. [27]

As has been said, Husserlian phenomenology exerted a considerable philosophical influence on Weyl.. In Weyl's work of 1918-22 we find much evidence of the great influence Husserl's ideas had on Weyl's philosophical outlook—one need merely glance through the pages of *Space-Time-Matter* or *The Continuum* to see it. Nevertheless a reading of *Insight and Reflection* shows Weyl to have moved away from the phenomenology which, as he remarks, "led me out of positivism once more to a freer outlook on the world." This divergence can in fact already be detected in Weyl's *The Open World* of 1932, in which, while granting that

> *The beginning of all philosophical thought is the realization that the perceptual world is but an image, a vision, a phenomenon of our consciousness; our consciousness does not directly grasp a transcendental real world which is as it appears…The postulation of the real ego, of the thou and of the world, is a metaphysical matter, not judgment, but an act of acknowledgment and belief,*

he continues:

> *It was an error of idealism to assume that the phenomena of consciousness guarantee the reality of the ego in an essentially different and somehow more certain manner than the reality of the external world; in the transition from consciousness to reality the ego, the thou and the world rise into existence indissolubly connected and, as it were, at one stroke*. (Weyl, *Open World,* 26–27).

I think we may take it that Weyl's use of the term "idealism" here is intended to include Husserl's phenomenology, since in *Insight and Reflection* Weyl remarks, in connection with Fichte's philosophy, that "Metaphysical idealism, toward which Husserl's phenomenology was then shyly groping, here received its most candid and strongest expression."

In *Insight and Reflection* Weyl describes Husserl as "an adversary of the psychologism which prevailed at the turn of the century", who went on to develop

> *the method of phenomenology, whose goal it was to capture the phenomena in their essential being—purely as they yield themselves apart from all genetical and other theories in their encounter with our consciousness. This quintessential examination unfolded to him a far broader field of evidently a priori insights than the twelve principles which Kant had posited as the constituting foundation of the world of experience.*

Weyl quotes a number of passages from Husserl's *Ideas*, which he calls his "great work of 1922." But certain of Weyl's comments on these passages have a somewhat critical tenor. For example, Weyl says

> *To point up the antithesis between an accidental, factual law of nature and a necessary law of being, Husserl cites the following two statements: "All bodies are heavy" and "All bodies have spatial extent." Perhaps he is right, but one senses even in this first example how uncertain generally stated epistemological distinctions become as soon as one descends from generality to specific concrete applications.*

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[27] Weyl [1946], 13.

He gives his own view on this issue in the form of a quotation from his own *Space-Time-Matter*:

> *The investigations about space that have been conducted [here] seem to me a good example of the kind of the kind of analysis of the modes of existence which is the object of Husserl's phenomenological philosophy, an example that is typical of cases in which we are concerned with non-immanent modes. The historical development of the problem of space teaches how difficult it is for us human beings entangled in external reality to reach a definite conclusion. … Certainly, once the true point of view has been adopted reason becomes flooded with light, and it recognizes and appreciates what is of itself intelligible to it,. Nevertheless, although reason was, so to speak, always conscious of this point of view in the whole development of the problem, it had not the power to penetrate it in one flash. This reproach must be directed at the impatience of those philosophers who believe that, on the basis of a single act of exemplary concentration, they are able to give an adequate description of being. In principle they are right, yet from the point of view of human nature, how utterly they are wrong! The example of space is at the same time most instructive with regard to the particular question of phenomenology that appears to me the decisive one: To what extent does the limitation of those aspects of being which are finally revealed to consciousness express an innate structure of what is given, and to what extent is this a mere matter of convention?*

He goes on to say:

> *Einstein's development of the general theory of relativity, and of the law of gravity which holds in the theory's framework, is a most striking confirmation of this method which combines experience based on experiments, philosophical analysis of existence, and mathematical construction. Reflection on the meaning of the concept of motion was important for Einstein, but only in such a combination did it prove fruitful.*

From this passage I think it may be inferred that Weyl had come to hold the view that the ultimate secrets of Being cannot be arrived at by philosophical reflection alone.

Weyl next turns to what he identifies as the central theme in Husserl's work, namely,

> *the relationship between the immanent consciousness, the pure ego from which all actions emanate, and the real psychophysical world, upon whose objects these acts are intentionally directed.*

Weyl characterizes Husserl's view of space as an object for the ego as follows:

> *Concerning space as an object, Husserl says that, with all its transcendence, it is something that is perceived and given in material irrefutability to our senses. Sensory data, "shaded off" in various ways within the concrete unity of this perception and enlivened by comprehension, fulfill in this manner their representative "function"; in other words, they constitute in unison with this quickened comprehension what we recognize as "appearances of" color, form, etc.*

However, Weyl quickly questions this account of the matter:

> *I do not find it easy to agree with this. At any rate, one cannot disavow that the particular manner in which, through this function of inspiration, an identifiable object is placed before me, is guided by a great number of earlier experiences…*

> *The theoretical-symbolic construction, through which physics attempts to comprehend the transcendental content behind the observations, is far from inclined to stop with this corporeally manifested identity. I should, therefore, say that Husserl describes but one of the levels which has to be passed in the endeavor through which the external world is constituted.*

Later Weyl appears to be somewhat uncomfortable with Husserl's epistemological idealism:

> *Concerning the antithesis of experience and object, Husserl claims no more than merely phenomenal existence for the transcendental as it is given in its various shadings, in opposition to the absolute existence of the immanent; i.e., the certitude of the immanent in contrast to the uncertainty of the transcendental perception. The thesis of the world in its accidental arbitrariness thus stands face to face with the thesis of the pure I and the I-life which is indispensable and, for better or worse, unquestionable. "Between awareness and reality there yawns a veritable chasm of meaning," he says,. "Immanent existence has the meaning of absolute being which* 'nulla re indiget ad existendum'; *on the other hand the world of the transcendental* 'res*' is completely dependent on awareness,—dependent, moreover, not just on being logically thinkable but on actual awareness."*

This brings Weyl to the enigma of personal identity, a problem to which he ascribes paramount importance:

> *Here finally arises in its full seriousness the metaphysical question concerning the relation between the one pure I of immanent consciousness and the particular lost human being which I find myself to be in a world full of people like me (for example, during the afternoon rush hour on Fifth Avenue in New York). Husserl does not say much more about it than that "only through experience of the relationship to the body does awareness take on psychological reality in man or animal."*

In this connection it is worth quoting what Weyl had to say concerning this issue in his *Address on the Unity of Knowledge,* delivered not long before.

> *…it is time now to point out the limits of science. The riddle posed by the double nature of the ego certainly lies beyond those limits. On the one hand, I am a real individual man…carrying out real and physical and psychical acts, one among many. On the other hand, I am "vision" open to reason, a self-penetrating light, immanent sense-giving consciousness, or however you may call it, and as such unique. Therefore I can say to myself both: "I think, I am real and conditioned" as well as "I think, and in my thinking I am free." More clearly than in the acts of volition the decisive point in the problem of freedom comes out, as Descartes remarked, in the theoretical acts. Take for instance the statement 2 + 2 = 4; not by blind natural causality, but because I* see *that 2 + 2 = 4 does this judgment as a real psychic act form itself in me, and do my lips form these words; two and two make four. Reality or the realm of Being is not closed, but open toward Meaning in the ego, where Meaning and Being are merged in indissoluble union—though science will never tell us how. We do not see through the* real *origin of freedom.*
>
> *And yet, nothing is more familiar and disclosed to me than this mysterious "marriage of life and darkness", of self-transparent consciousness and real being that I am myself. The access is my knowledge of myself from within, by which I am aware of my own acts of perception, thought, volition, feeling and doing, in a manner entirely different from the theoretical knowledge that represents the "parallel"*

> *cerebral process in symbols. The inner awareness of myself is the basis for the more or less intimate understanding of my fellow-men, whom I acknowledge as beings of my own kind. Granted that I do not know of their consciousness in the same manner as my own, nevertheless my "interpretative" understanding of it is apprehension of indisputable accuracy. As hermeneutic interpretation it is as characteristic for the historical, as symbolic construction is for the natural, sciences. Its illumining light not only falls on my fellow-men; it also reaches, though with ever-increasing dimness and incertitude, deep into the animal kingdom. Kant's narrow opinion that we can feel compassion, but cannot share joy, with other living creatures, is justly ridiculed by Albert Schweitzer who asks: "Did he ever see a thirsty ox coming home from the fields drink?" It is idle to disparage this hold on nature "from within: as anthropomorphic and elevate the objectivity of theoretical construction, though one must admit that understanding, for the very reason that it is* concrete *and* full*, lacks the freedom of the "hollow symbol". Both roads run, as it were, in opposite directions: what is darkest for theory, man, is the most luminous for the understanding from within; and to the elementary inorganic processes, that are most easily approachable by theory, interpretation finds no access whatsoever.*

Returning to *Insight and Reflection,* Weyl goes on to compare Husserl's position with that of Fichte, a philosopher whose views Weyl says also had a pronounced influence on him. Although Weyl claims to find "preposterous" the actual details of what he calls Fichte's "constructivism", according to which the world is a necessary construction of the ego, nevertheless we find him asserting that

> *in the antithesis of constructivism and phenomenology, my sympathies lie entirely on [Fichte's] side.*

But he quickly adds:

> *yet how a constructive procedure which finally leads to the symbolic representation of the world, not a priori, but rather with continual reference to experience, can really be carried out, is best shown by physics—above all in its two most advanced stages: the theory of relativity and quantum mechanics.*

Soon afterwards Weyl introduces a geometric analogy which, he believes,

> *will be helpful in clarifying the problem with which Fichte and Husserl are struggling, namely, to bridge the gap between immanent consciousness which, according to Heidegger's terminology, is ever-mine, and the concrete man that I am, who was born of a mother and who will die.*

In this analogy objects, subjects, and the appearance of an object to a subject are correlated respectively with *points on a plane, (barycentric) coordinate systems in the plane,* and *coordinates of a point with respect to a such a coordinate system.*

In Weyl's analogy, a coordinate system S consists of the vertices of a fixed nondegenerate triangle T; each point p in the plane determined by T is assigned a triple of numbers summing to 1—its *barycentric coordinates* relative to S—representing the magnitudes of masses of total weight 1 which, placed at the vertices of T, have centre of gravity at p. Thus objects, i.e. points, and subjects i.e., coordinate systems or triples of points belong to the same "sphere of reality." On the other hand, the *appearances* of an object to a subject, i.e., triples of numbers, lie, Weyl asserts, in a different sphere, that of *numbers*. These *number-appearances,* as Weyl calls them, correspond to the experiences of a subject, or of pure consciousness.

From the standpoint of naïve realism the points (objects) simply exist as such, but Weyl indicates the possibility of constructing geometry (which under the analogy

corresponds to external reality) solely in terms of number-appearances, so representing the world in terms of the experiences of pure consciousness, that is, from the standpoint of idealism. Thus suppose that we are given a coordinate system *S*. Regarded as a subject or "consciousness", from its perspective a point or object now corresponds to what was originally an appearance of an object, that is, a triple of numbers summing to 1; and, analogously, any coordinate system *S′* (that is, another subject or "consciousness") corresponds to three such triples determined by the vertices of a nondegenerate triangle. Each point or object *p* may now be *identified* with its coordinates relative to *S*. The coordinates of *p* relative to any other coordinate system *S′* can be determined by a straightforward algebraic transformation: these coordinates represent the *appearance* of the object corresponding to *p* to the subject represented by *S′*. Now these coordinates will, in general, *differ* from those assigned to *p* by our given coordinate system *S*, and will in fact coincide for all *p* if and only if *S′* is what is termed by Weyl the *absolute* coordinate system consisting of the three triples (1,0,0), (0,1,0), (0,0,1), that is, the coordinate system which corresponds to *S* itself. Thus, for this coordinate system, "object" and "appearance" coincide, which leads Weyl to term it the *Absolute I*. [28]

Weyl points out that this argument takes place entirely within the realm of numbers, that is, for the purposes of the analogy, the *immanent consciousness*. In order to do justice to the claim of objectivity that all "I"s are equivalent, he suggests that only such numerical relations are to be declared of interest as remain unchanged under passage from an "absolute" to an arbitrary coordinate system, that is, those which are invariant under arbitrary linear coordinate transformations. According to Weyl,

> *this analogy makes it understandable why the unique sense-giving I, when viewed objectively, i.e., from the standpoint of invariance, can appear as just one subject among many of its kind.*

Then Weyl adds an intriguing parenthetical observation:

> *Incidentally, a number of Husserl's theses become demonstrably false when translated into the context of the analogy—something which, it appears to me, gives serious cause for suspecting them.*

Unfortunately, we are not told precisely which of Husserl's theses are the "suspect" ones.

Weyl goes on to emphasize:

> *Beyond this, it is expected of me that I recognize the other I—the* you*—not only by observing in my thought the abstract norm of invariance or objectivity, but absolutely: you are* for you*, once again, what* I *am for* myself: *not just an existing but a conscious carrier of the world of appearances.*

This recognition of the *Thou,* according to Weyl, can be presented within his geometric analogy only if it is furnished with a purely *axiomatic* formulation. In taking this step Weyl sees a third viewpoint emerging in addition to that of realism and idealism, namely, a *transcendentalism* which "postulates a transcendental reality but is satisfied with modelling it in symbols."

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[28] This phrase Weyl derives from Fichte, whom he quotes as follows:

> *The I demands that it comprise all reality and fill up infinity. This demand is based, as a matter of necessity, on the idea of the infinite I; this is the absolute I (which is not the I given in real awareness.*

But Weyl, ever-sensitive to the claims of subjectivity, hastens to point out that this scheme by no means resolves the enigma of selfhood. In this connection he refers to Leibniz's attempt to resolve the conflict between human freedom and divine predestination by having God select for existence, on the grounds of sufficient reason, certain beings, such as Judas and St. Peter, whose nature thereafter determines their entire history. Concerning this solution Weyl remarks

> *[it] may be objectively adequate, but it is shattered by the desperate cry of Judas: Why did* I *have to be Judas! The impossibility of an objective formulation to this question strikes home, and no answer in the form of an objective insight can be given. Knowledge cannot bring the light that is I into coincidence with the murky, erring human being that is cast out into an individual fate.*

Weyl's divergence from pure phenomenology is made evident by the passage immediately following, which shows him to have come to embrace a kind of theological existentialism:

> *At this point, perhaps, it becomes plain that the entire problem has been formulated up to now, and especially by Husserl, in a theoretically too one-sided fashion. In order to discover itself as intelligence, the I must pass, according to Descartes, through radical* doubt*, and, according to Kierkegaard, through radical* despair *in order to discover itself as existence. Passing through doubt, we push through to knowledge about the real world, transcendentally given to immanent consciousness. In the opposite direction, however—not in that of the created works but rather in that of the origin—lies the transcendence of God, flowing from whence the light of consciousness—its very origin a mystery to itself—comprehends itself in self-illumination, split and spanned between subject and object, between meaning and being.*

Weyl says that from the late works of Fichte he moved to the teachings of Meister Eckhart, whom Weyl calls the "deepest of the Occidental mystics", the originality of whose basic religious experience cannot be doubted:

> *It is the inflow of divinity into the roots of the soul which he describes with the image of the birth of the "Son" or of the "Word" through God the Father. In turning its back on the manifold of existence, the soul must not only find its way back to this arch-image, but must break through it to the godhead that lives in impenetrable silence.*

It was through the reading of Eckhart that Weyl "finally found for himself the entrance to the religious world." But he admits that his metaphysical-religious speculations never achieved full clarity, adding that "this may perhaps also be due to the nature of the matter."

In his later years, Weyl says,

> *I did not remain unaffected either by the great revolution which quantum physics brought about in natural sciences, or by existentialist philosophy, which grew up in the horrible disintegration of our era. The first of these cast a new light on the relation of the perceiving subject to the object; at the center of the latter, we find neither a pure I nor God, but man in his historical existence, committing himself in terms of his existence.*

*

During his long philosophical voyage Weyl stopped at several ports of call: in his youth,

Kantianism and positivism; then phenomenological idealism; and finally a kind of theological existentialism. But apart from his brief flirtation with positivism (itself, as he says, the result of a disenchantment with Kant's "bondage to Euclidean geometry") Weyl's philosophical orientation was in its essence idealistic: he cleaved always to the primacy of intuition that he had first learned from Kant, and to the centrality of the individual consciousness that he first absorbed from Fichte and Husserl. But while he continued to admire Husserl's philosophy, I infer from his remarks in *Insight and Reflection* that he came to regard it as lacking in two essential respects: first, it failed to give due recognition to the transcendental external world, with which Weyl, in his capacity as a natural scientist, was concerned; and secondly, and perhaps in Weyl's view even more seriously, that it failed to deal adequately with the enigma of selfhood: the fact that I am the person I am. Grappling with the first problem led Weyl to emphasize the essential importance of symbolic construction in grasping transcendental external reality, a position which brought him close to Cassirer in certain respects; while the second seems to have led him to existentialism and even to religious mysticism.

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