

The bibliography paradox revisited

John L. Bell

There is a well-known version of Russell's paradox concerning the *bibliography of all bibliographies which fail to list themselves*. The usual analysis of this paradox leads to the conclusion that such a bibliography is self-contradictory and so therefore cannot exist. However, as we show, a more searching analysis leads to a rather different conclusion.

Let us suppose that that we are given two domains of things: *titles* and lists of titles, or *bibliographies*. We shall also suppose that each bibliography has been assigned a (unique) title. Now consider the bibliography R which lists all and only titles of bibliographies which fail to list their own titles. Then R lists its own title; for if not, then its title, call it r , has the property that there is a bibliography, namely R itself, of which r is the title, and which fails to list r . But, since R lists all such titles, it must therefore list r . We conclude that R lists its own title after all. Now since R lists its own title r , it follows from the definition of R that r must also be the title of a bibliography S which *fails to list its own title*. Then, R and S have the same title (namely, r) but they cannot be the same, since R lists its own title but S does not. We conclude therefore that *there must exist two different bibliographies which are assigned the same title*. And as we have seen, one of these bibliographies is R , which certainly exists.

Note the difference between the conclusion here and that of Russell's paradox. In the latter the conclusion is that there is a plurality which cannot be treated as a single thing. In the bibliographic case the conclusion is that, while each bibliography, considered as a plurality of titles, can be assigned a title - that is, in a certain sense treated as an individual - *there will always exist two distinct pluralities which have to be treated as if they were the same thing*. In both cases there are more pluralities than there are individuals.

This analysis of the bibliography paradox is (of course) a lightly disguised version of Cantor's theorem that, for any set X , there can be no one-to-one mapping of the power set of X into X .