

The consolations of Zorn's lemma and the escape into maximality.

The form of the linear is simple, likely inescapable, both geometrically and perceptually, and its discrete correlate, the serial ordering, is canonically exemplified by the natural numbers, an inevitable product of thinking. We are all trapped in a linearly ordered temporal structure, which, like gravity, we have come to accept as a necessary condition for our continued existence. But when the linear is extended beyond the existential to the social and psychological realms, it becomes a kind of tyranny. For instance, I have always regarded as objectionable the idea of "knowing one's place" – the acceptance of one's inferior (or, inversely, superior) position in a linearly ordered social structure. Now *that* order is extrinsic; it can be changed objectively, in principle at least, by social revolution, and assuaged psychologically by contemplating the idea of revolution, or by escaping into art or unconsciousness. But the *intrinsic*, the immanent mental form of linearity is much more difficult for the (conscious) psyche to overcome. How does one cope with the psychological pain of coming honestly to recognize one's own intrinsic inferiority to another? This potential blow to the ego is almost certain to be actualized given the fact that billions of others exist who are essentially the same as – and so comparable – with oneself, and some of these are bound, on purely statistical grounds, to possess abilities which are superior, on objective grounds, to one's own. For the true stoic, that insight alone should suffice to give repose. In "Brave New World" Aldous Huxley provided another answer: chemically impose, and then celebrate, the linearity of human abilities ("thank Ford I'm a Beta"). But I stubbornly reject the idea of a linear scale of superiority and inferiority within which, by implication, there is room for just one "best", or maximum. Indeed the pain experienced by the ego at not achieving that maximum can give way to the belief that one has been subjected to an actual affront: consider, for example, the idea of going "postal". To avoid such unpleasantness I find it agreeable to put the linear scale aside and replace it with a partial ordering containing incomparable elements. And in any case, don't we say of great artists or thinkers that they are "incomparable"? For example, aren't James Joyce and Tolstoy, or Bach and Mozart, or Einstein and Newton, or Euler and

Gauss, or Plato and Aristotle, or Laurel and Hardy, incomparable, and yet is not each, within his own creative sphere, unsurpassable? Each is a maximal element of the partially ordered set of achievers within their associated sphere of endeavour. I believe that Zorn's lemma can provide a psychological justification for the necessary existence of maximal achievers. For consider the set of all achievers in a particular sphere of endeavor, for instance the set of painters, the set of mathematicians, or of tennis players, or of violinists. It can be seen that this set, call it A , partially ordered by comparability of achievement, is inductive. For suppose given a chain X in A , i.e a subset X of A each member of which acknowledges his/her comparability with the rest. Now take any member P of X . There seem to be three natural possibilities: (1) P believes (whether justified or not) that *he* is (or could become) the best in X , (2) P acknowledges his inferiority to certain other members of X , and among the set of these identifies someone of supreme ability (this is often encountered among mathematicians) (3) P acknowledges his inferiority to certain other members of X , but cannot abide the idea that those others do not have to suffer the feelings of inferiority that *he/she* has to endure. In this case P finds consolation by *universalizing* his/her acknowledged inferiority by *imagining* the existence of an individual - a "god" - superior to every member of X (and so necessarily not a member of X , but, like the gods of ancient Greece, still a member of A). Thus every chain in A has an upper bound, A is accordingly inductive, the premises of Zorn's Lemma are satisfied in the human sphere, and the existence of maximal elements therein confirmed.

Of course, quite independently of these fanciful speculations, it would be absurd to compare oneself with the likes of Joyce or Tolstoy, to say nothing of Laurel and Hardy. Yet it is still consoling that, by abandoning the harshness of the linear scale, one can find a niche within which one is maximal, floating in a cool calm Hindemithian pool so different from Schoenberg's turbulent boiling ocean.