Googol Plus One: A Reminiscence

A number of years ago my friend John Mayberry remarked to me that good mathematicians occasionally have failures of insight. He then described his favourite example, which appears in Hilary Putnam's well-known paper *Mathematics without Foundations.* In that paper Putnam states:

If the whole physical universe is thoroughly finite, both in the large and in the small, then the statement "$10^{100} + 1$ *is a prime number" may be one whose truth value we can never know.*

John pointed out, with considerable glee, that it's easy to see, and Putnam should have seen it, that simple algebra shows that $10^{100} + 1$ (googol plus 1) is not prime. For from the algebraic fact that $x^5 + 1$ is divisible by $x + 1$ and the fact that $10^{100} + 1 = (10^{20})^5 + 1$, it follows that $10^{100} + 1$ is divisible by $10^{20} + 1$. I was left with considerable admiration, mixed with a certain envy, for my friend's insight. John wondered whether this fact had later occurred to Putnam, or whether it had been pointed to him by any of his colleagues. It only struck me many years later that John's glee at Putnam's slip was probably also fueled by the title of the paper, *Mathematics without Foundations,* in which it appears. For John, mathematics without foundations was a kind of absurdity, not to be taken seriously.

I had never met Putnam, but I had long admired him. As a mathematical logician I was aware of his contributions to the (negative) resolution of Hilbert's 10^{th} Problem. I had also been amused and impressed by his appearance on the 1978 BBC television program *Men of Ideas* in which various philosophers (including Marcuse, Chomsky, Quine, Ayer as well as the honorary "man", the novelist and philosopher Iris Murdoch) engaged in conversation with Bryan Magee, a principled but somewhat pedantic English philosopher. To create an atmosphere of informality, the participants were seated at the ends of a long sofa. In striking contrast with Bryan Magee's earnestness, I found

Putnam's easy, laid back style agreeable, entirely compatible with the comfortable appearance of the sofa on which he relaxed so casually.

Years later it happened that Putnam was invited to speak at a conference organized in honor of my colleague Bill Demopoulos to be held at my university in Canada. I had been invited to the reception arranged for the speakers, and Bill, who knew Putnam quite well, was happy to introduce me to him. Naturally I figured that this presented an ideal opportunity for me to ask Putnam whether he had come to realize that googol plus one (*gpo*) is actually composite. Reflecting on how I should put this question to him, it occurred to me to try to identify some smaller factors of *gpo*. John had already observed that *gpo* is divisible by $10^{20} + 1$. Then it struck me that since $10^{20} + 1 = (10^4)^5 + 1$, $10^{20} + 1$ is itself divisible by $10^4 + 1$, i.e. 10001. Lacking the numerical insight of a Ramanujan, I could not immediately tell whether 10001 was composite, but I soon found that it is in fact the product of 73 and 137. I was delighted that the latter happens to be the English astrophysicist Eddington's magic number, which figures prominently in the neo-Pythagorean cosmology expounded in his *Fundamental Theory,* and Eddington's obsession with which is satirized in Bertrand Russell's *The Mathematician's Nightmare.*

Armed with this revelation, I felt that when I finally met Putnam, I would not merely be posing John Mayberry's original question but would have something original to say. So soon after Putnam and I were introduced, I launched straight into the matter, first asking him whether he recalled his remark about *gpo* in his old paper, to which he responded in the affirmative, and then whether he had later become aware that *gpo* was composite. Somewhat surprised, probably, at the interrogational turn our conversation was taking (and also likely wondering whether he had been introduced to a lunatic), he replied that, no, he hadn't. Then I went on to present my *pièce de résistance,* the procedure showing that *gpo* is divisible by Eddington's magic number. I shall never forget Putnam's response, both witty and gracious, *Well, you can't win 'em all!*