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Bell, John L. *** 26014**
A primer of infinitesimal analysis.
Cambridge: Cambridge University Press, (ISBN 0-521-62401-0/hbk). xiii, 122 p. £ 19.95; $ 29.95 (1998).

In this book, the author presents in a clear and well-written manner a theory of "smooth infinitesimal calculus". The word "infinitesimal" indicates that the theory must involve a mathematically sound concept of an infinitesimal quantity.

In 1961, *Abraham Robinson* showed in his ground breaking paper entitled "Non-standard analysis" [Nederl. Akad. Wet., Proc., Ser. A 64, 432–440 (1961; Zbl 102.00708)] that the statement: "There exists non-zero infinitesimals" is consistent in a precise formalized sense with the axioms of the real number system. This observation led to a rigorous foundation of the calculus based on Robinson's concept of infinitesimals.

The infinitesimals referred to in the title of the book have, however, a different origin. About a decade after Robinson's discovery a new development in category theory took place that gave rise to the theory of toposes. A topos is in a sense a kind of category that possesses an internal structure that is sufficiently rich to carry out all of the usual constructions of mathematics. Associated with toposes are mathematical languages which have a common logic. This logic turs out to be the intuitionistic logic or constructive logic. In an intuitionistic logic all principles of classical logic are valid except those that depend on the law of excluded middle. It is most remarkable that toposes can be defined, called smooth toposes, in which infinitesimal objects exist with vanishing squares. In such smooth worlds all the quantities infinitesimal as well as static undergo smooth variations.

Chapters 1 and 2 are devoted to a description of the basic aspects of smooth worlds and it is illustrated how the development of the calculus with the square zero infinitesimals can be accomplished. Chapters 3 and 4 are devoted to applications of the differential calculus in smooth infinitesimal analysis to topics in geometry and physical problems. Chapter 5 takes up the smooth differential calculus of several variables. Integration and related topics are treated in Chapter 6. Differential geometry in smooth analysis is the subject of Chapter 7. An axiomatic approach to smooth analysis is given in Chapter 8. It also presents a very clear and very readable account in which sense the two new theories of infinitesimals, one based on nonstandard models and classical logic by Robinson and the other based on smooth toposes, are related. In the appendix to the book, the author presents a sketch of the construction of models for smooth infinitesimal analysis.

The author is to be congratulated for his effort to make this important alternative theory of infinitesimals available to the general reader. The book is warmly recommended to anyone that works in analysis and geometry. *W. A. J. Luxemburg (Pasadena).*