Discussions

THE INFINITE PAST REGAINED: A REPLY TO WHITROW *

In his [1978], Whitrow attempts to prove *a priori* that there cannot exist an infinite, temporally ordered sequence of past events. Following Kant, he claims that if there were such a sequence, the present could never have been reached; and his argument for this claim relies on a version of the so-called Tristram Shandy argument. In this note I want primarily to show two things. First, that Whitrow's version of the Tristram Shandy argument is circular in the sense that it is only valid if the conclusion is included as one of the premises. And secondly, even if the conclusion is granted, it does not do the job Whitrow requires of it.

Let me begin by reconstructing Whitrow's version of the Tristram Shandy argument (which I shall usually refer to simply as the Tristram Shandy argument, without further qualification). It concerns the existence of an infinite sequence of discrete *future* events. What Whitrow wants to show is that there can be no event which is infinitely far away in the future. Thus, starting with an initial event E_0 (the 'present'), if we have an infinite sequence of discrete events $E_0, E_1, \ldots, E_n, \ldots$ where, for $m < n$, E_m occurs before E_n, Whitrow claims that it is *a priori* impossible for there to occur a 'limit' event E_ω which follows *all* the E_n, even when the events are not necessarily 'evenly spaced' in time. The Tristram Shandy argument for this claim runs as follows. Suppose it takes Tristram Shandy one year to commit to paper the events of a single day. Then by the end of the *n*th year he will have recorded the events of the first *n* days. Thus, the lag between an event and the future time at which it is recorded tends to infinity with time. Hence, there can be no 'limit' event at whose start *all* preceding events have already been recorded. In other words, if Tristram Shandy lives forever, each event in his life *will be* recorded, but there will be no time at which all preceding events *have been* recorded.[1]

Let us consider this argument more closely. First, it must assume, contrary to Whitrow's claim,[2] that the events involved are 'evenly spaced' in time, or, at least, that the intervals between successive events do not tend to zero with time. For it is logically possible for there to exist a writer who records events at exactly the rate at which they occur. By the end of a given day this writer will have recorded the events of that day. Following Zeno, we subdivide the

* I am indebted to Mike Hallett for valuable criticisms and suggestions.

[1] Whitrow names Russell [1937] as his source for the Tristram Shandy argument. However, it should be emphasised that Russell's version of the argument is not the same as Whitrow's. For Russell is only concerned to point out the 'paradoxical' fact that, if Tristram Shandy lives forever, each event in his life will be recorded, although the lag between an event and the time at which it is recorded increases indefinitely. Russell (in my view, correctly) makes no attempt to employ this 'paradox' to prove the impossibility of infinite past or future, nor does he say (although Whitrow attributes this to him) that a time would eventually arrive when all the days of Tristram Shandy's life would be written about.

[2] Whitrow [1978], p. 42.

day into decreasing temporal intervals $I_1, I_2, \ldots$ where I_1 is the first half day, I_2 is the next quarter day, ..., I_n is the next ($1/2^n$th) day, *etc.*, and select, for each n, an event E_n in the interval I_n. Then by the end of the day the events E_n have all been recorded. Thus, in this case we can take the beginning of the next day as a 'limit' event before which all the preceding events E_n have been recorded. The existence of such a writer would then constitute a counter-example to Whitrow's argument, *unless* it presupposes that the interval between successive events do not tend to zero with time.

This point, however, is not particularly important. Let us merely amend Whitrow's argument by assuming that the sequences of events involved are all such that the intervals between successive events do not tend to zero with time and proceed from there. Call such sequences of events *proper* sequences. My first main claim is this: Whitrow's argument for the non-existence of events E_ω is *circular*. To begin with, it is clear that the *most* the Tristram Shandy argument establishes is the following, which for convenience I will call the *Tristram Shandy thesis*: *no proper temporally increasing sequence of events can have order type* $\omega+1$. Recall that $\omega+1$ is the order type (in this case, an ordinal) of the ordered set obtained by appending a 'last' or 'limit' element to the ordered set ω of natural numbers. Now, I claim that the Tristram Shandy argument given by Whitrow *assumes* the truth of the Tristram Shandy thesis, so there is a logical circle involved. To see this, let us list the stages in the argument.

(1) By the end of the nth year the events of the first n days have been recorded;

(2) the lag between an event and the future time at which it is recorded tends to infinity with time (more precisely, as $364n$, where n is the number of days elapsed);

(3) there is no 'limit' event at whose start all preceding events have been recorded.[1]

It is obvious that (1) implies (2). But observe that (2) *only* refers to events E_n which are *finitely accessible* from the initial event E_0. Statement (2) asserts *nothing whatsoever* about any possible 'limit' event following all the E_n. In fact, from (2) we can only draw the conclusion that

(3′) there is no event E_n $(n > 0)$ *finitely accessible* from E_0 at whose start all preceding events have been recorded.

But evidently statement (3′) is not the same as, nor does it imply, statement (3). Indeed, if we want to obtain (3), we have to *assume* that all future events are finitely accessible from the initial one, which is of course just another way of expressing the Tristram Shandy thesis, *i.e.* exactly what I contend the argument purports to prove.

To make the point clearer, let us replace the sequence of events E_n by the sequence ω of natural numbers, and see how the argument fares there. Then (1), (2), (3) become:

(*i*) Consider the function $f(x) = 365x$;

(*ii*) $f(n) - n$ tends to infinity as the natural number n tends to infinity;

(*iii*) for any number $x > 0$, we have $f(x) > x$.

[1] We ignore the trivial 'limit' event E_0.

Now it is clear that (*ii*) follows from (the definitional) (*i*); but (*iii*) only holds *provided that by 'number' we mean 'finite (natural) number'*. If we allow *infinite ordinal numbers* as values for x—and there is no *a priori* reason why we should not—then (*iii*) is *false*: for it is well known that $f(\omega) = \omega$, where ω is the first infinite ordinal. Thus, in order to be able to assert the truth of (*iii*), we have to *assume that infinite ordinals have been excluded*. By itself (*ii*) cannot yield (*iii*) and *a fortiori* cannot yield the conclusion that there are no infinite ordinals. This is exactly the same situation as that arising in the Tristram Shandy argument: in order to draw the conclusion you have to assume it to begin with!

My second point is that even if the Tristram Shandy thesis is adopted *axiomatically*[1] it fails to establish what Whitrow wants to establish about the *past*. Whitrow claims that his Tristram Shandy argument also shows that there can be no infinite sequence of preceding past events ('evenly spaced' in time). But it seems to me that this claim is simply wrong: at any rate, the Tristram Shandy thesis, as formulated above, implies no such thing. To see this, let us observe that such a (temporally ordered) sequence of past events would have the order type ω^* of the set of negative integers with its natural ordering. Now plainly such a sequence does not have order type $\omega+1$ (or even order type ω) and neither does any of its subsequences. Thus, no violation of the Tristram Shandy thesis arises from the assumption of the existence of an infinite past sequence (of order type ω^*). Accordingly, even granting the Tristram Shandy thesis as an *axiom* does not suffice to prove the impossibility of an infinite past. Indeed, I agree with Popper's [1978] remark:

> The attempt to show by *a priori* reasoning the impossibility of time without beginning seems to me doomed to failure.

In my opinion, the main source of confusion in Whitrow's argument is his failure to distinguish between infinite *sets* and infinite *order types*, and to realise that there is always more than one of the latter corresponding to any given one of the former. For example, when considering an infinite sequence E_0, E_{-1}, $E_{-2}, \ldots, E_{-n}, \ldots$ of events going into the past, he says:

> If n events occurred in the sequence before E_0, then there must have occurred an event designated E_{-n} in my notation. *Similarly, if aleph-zero events occurred before E_0, then there must actually have occurred (in time past) events $E_{-\alpha}$.*[2]

Here by $E_{-\alpha}$ is meant an event preceding all the events E_{-n}. But there are no grounds whatsoever for drawing the italicised conclusion. Although the *set* of events $\{E_0, E_{-1}, E_{-2}, \ldots\}$ is infinite, its *order type* is ω^* and so it does not have a first member. The mere existence of such a sequence does not imply the further existence of an event *preceding* all its members; to draw this latter conclusion one requires stronger assumptions.

The same confusion arises further on. Whitrow says:

> Consequently, the concept of an infinite sequence of past events is incapable of culminating in the present event.[3]

[1] One can argue for the plausibility of this thesis, *cf.* below, *p.* 165.
[2] *Op. cit.*, p. 43, my italics.
[3] *Op. cit.*, p. 43.

Now this assertion would undoubtedly be correct if, for example, the sequence in question were to have the order type $1+\omega^*$ of the set obtained by appending a 'first' element to the sequence ω^* of negative integers (or indeed any order type of the form $\alpha+\omega^*$, where α is an ordinal > 0). But the assertion is evidently *false* when the sequence has order type ω^*. For then each event in the sequence will be of the form E_{-n}, where n is a natural number, and starting from there we will duly arrive at the present in precisely n steps. In this case there is no 'first' event from which it would take infinitely many steps to reach the present.

The confusion is further compounded when, in a footnote, he asserts:

> There is symmetry, in this respect, of future and past: an infinite sequence (actual infinity, or aleph-zero) of past events corresponds to an actually infinite (aleph-zero) sequence of future events. The latter concept has to be rejected and likewise, in my view, the former.[1]

Now the first assertion in this quotation is simply incorrect. For even if there is an infinite sequence of past events (of order type ω^*, say), it does not follow that from this one can construct an infinite sequence of future events. To do this one would have to extract a subsequence of order type ω, and, as we have observed above, no sequence of order type ω^* has such a subsequence. In fact, it is easy to see that in this respect the only conclusion we can draw from the existence of a sequence of past events of order type ω^* is that there *have been* arbitrarily long *finite* sequences of future events; we *cannot infer that* there is, or has been, an infinite sequence of future events.[2] On the other hand, and also contrary to the opinion expressed in the above quotation, we have seen that there is nothing at all contradictory in assuming the existence of an infinite proper sequence of future events of order type ω. Furthermore, even the Tristram Shandy thesis which we have agreed to grant Whitrow does not imply the non-existence of such a sequence: for the thesis asserts the non-existence of a proper sequence of order type $\omega+1$, but *says nothing* about the existence or not of sequences of order type ω.

It therefore seems to me that, as with the case of an infinite past, it is equally impossible to prove *a priori* the non-existence of an infinite future.[3]

Let me close with a few remarks on the plausibility of the Tristram Shandy thesis, and a certain generalisation of it. We have seen that it does not preclude the existence of infinite *sets* of events, but only bars them from having certain *order types* (like $\omega+1$) in their temporal ordering. A natural generalisation of the thesis, which I shall call the *finite accessibility hypothesis*, is the assertion that each event be finitely accessible (in time) from every other event. This hypothesis not only excludes 'future' order types such as $\omega+1$ but also 'past' order types such as $1+\omega^*$. If the finite accessibility hypothesis is true (and

[1] *Op. cit.*, p. 43.

[2] Similarly, from the existence of a sequence of future events of order type ω we can only draw the conclusion that there *will be* arbitrarily long *finite* sequences of past events; we *cannot* infer that there is, or will be, an infinite sequence of past events.

[3] Of course, this does not preclude the possibility of establishing these assertions on the basis of some *physical* theory (*e.g.* general relativity).

there is no first or last moment), then discrete linear time[1] has the order type $\omega^* + \omega$ of the set of negative and positive integers. If, on the other hand, it is *false* (and there is no first or last moment) then discrete linear time can be split up into a plurality of disjoint segments each of order type $\omega^* + \omega$. Each segment then corresponds to a 'state of the universe' of infinite duration. Under these conditions, however, no event in a given segment is finitely accessible (in time) from an event in any other segment. Thus from the standpoint of an observer stationed in a given segment, the objective existence of any *other* segment must seem highly dubious. It would therefore seem reasonable for such an observer (*e.g.* oneself) to 'confine attention' to the segment in which he is situated, and thereby *accept* the finite accessibility hypothesis (and, *a fortiori*, the Tristram Shandy thesis) as a plausible assumption—albeit not provable *a priori*—about discrete linear time. Even with this hypothesis, however, it is still logically possible for time to have neither a beginning nor an end.

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